Consumer Surplus with Incomplete Markets: Applications to Savings and Microfinance

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Abstract

The household welfare gains from financial inclusion are empirically elusive. I establish that household welfare gains from a financial technology are equal to the area under dynamically compensated demand in a household model with incomplete financial markets, and general technology, preferences, and choice sets. I then estimate compensated demand for financial technologies leveraging three randomized control trials that introduce experimental variation in interest rates. Welfare gains per dollar lent or saved are small as compensated demand elasticities are large, but still correspond to large aggregate welfare gains from financial inclusion.

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1 Introduction

What are the household welfare gains from microfinance? More than 200 million households borrow from microfinance institutions annually. Despite this scale, a growing empirical consensus holds that microfinance has failed to meet its original promise, instead offering positive but modest impacts on borrowers (Banerjee et al., 2015b; Meager, 2019). However, estimating welfare gains is difficult. Households may benefit from microfinance through a broad range of mechanisms – microfinance enables productive investments and lumpy purchases (Banerjee et al., 2021), consumption smoothing (Lane, 2020), and acts as a gateway to traditional credit (Rigol & Roth, 2021). As a result, the sign and magnitude of impacts of microfinance on common welfare proxies, including household consumption, may be inconsistent with impacts on welfare (Banerjee et al., 2015b), and theory accounting for these diverse mechanisms is necessary to quantify impacts on welfare (Karlan & Zinman, 2019).

In this paper, I propose a method to estimate the household welfare gains from financial technologies, including microfinance, that builds on the following result: consumer surplus is equal to the area under dynamically compensated demand. To capture the complexity of household consumption and investment decisions in developing countries, I develop a dynamic household model with incomplete financial markets and general technology, preferences, and choice sets. I show that the area under dynamically compensated demand is equal to consumer surplus. I then estimate compensated demand and consumer surplus for financial technologies leveraging experimental variation in prices from three randomized control trials – consumer surplus per dollar lent or saved is small, as compensated demand elasticities are large, but still corresponds to large aggregate consumer surplus from financial technologies.

The intuition underlying the area under compensated demand as a measure of consumer surplus from financial technologies is captured by the following stylized example, visualized in Figure 1: consider a borrower that takes a loan with $1000 principal, for which the borrower will need to make 12 monthly payments of $100. The borrower has a minimum “willingness-to-accept” (WTA) – the minimum acceptable principal, holding fixed monthly payments, at which the borrower would take the loan. The borrower’s surplus from the loan is simply the difference between the $1000 principal (the “price”) and their WTA – although the borrower discretely changes their behavior when the offered principal falls below their WTA, as the borrower declines the loan, an envelope theorem holds and these responses have no first order effects on welfare. In canonical consumer theory, demand for a good as a function of price, holding fixed product attributes, traces out willingness-to-pay; for loans, demand for monthly payments as a function of the offered principal traces out WTA. This
inverts the conventional perspective on loans: rather than considering quantities borrowed as a function of interest rates, which yields no natural measure of expenditures, I characterize demand for monthly payments as a function of “prices”, with negative loan amount as the natural measure of expenditures. As a consequence, the area under demand for the loan is equal to total borrower surplus, that is the difference between offered principal and WTA.

Figure 1: An example of loan demand and borrower surplus

I consider the problem of estimating consumer surplus in a general dynamic household model with incomplete financial markets. As financial markets are incomplete, households may be unable to transfer income across states, so “permanent income” is not well defined. Compensating transfers for a change in prices are therefore dynamic – by Roy’s identity, for any small price change, the change in prices times period-state-contingent quantities are compensating transfers (Farhi et al., 2022). Integrating these compensating transfers over a change in prices suggests period-state-contingent area under dynamically compensated demand is equal to consumer surplus.

I show that period-state-contingent area under dynamically compensated demand does in fact equal consumer surplus under weak technical assumptions on preferences, technology, and choice sets. This flexibility is important to capture important aspects of household finance in developing countries – households may have non-convexities in consumption and investment (Banerjee et al., 2021), may face dynamic and potentially non-pecuniary repayment incentives when making strategic default decisions (Feigenberg et al., 2013), and may respond to access to financial technologies both ex-ante and ex-post (Lane, 2020). I allow this flexibility by assuming households are rational, and applying an envelope theorem (Milgrom & Segal, 2002) by assuming that changes in prices enter the household problem only through the household’s dynamic budget constraint.
Estimation of dynamically compensated demand requires estimates of price elasticities and income effects; in contrast to the static household model, rather than a single income effect, the full matrix of intertemporal income effects on demand is necessary. I take a sufficient statistics approach (Chetty, 2009) and show this matrix of intertemporal income effects is an inner product of two terms – the intertemporal matrix of marginal propensities to spend out of temporary income, and growth in demand. Restrictions on this matrix yield two common frameworks: with complete financial markets, the intertemporal matrix of marginal propensities to spend is rank 1, while absent financial markets and with separable utility (i.e., the static household model), it is diagonal. In applications, I use empirical estimates of growth in demand, and I suggest theory-motivated bounds on intertemporal marginal propensities to spend.

A number of strong assumptions are embedded in my modeling framework. Common to many frameworks used to analyze the impacts of financial technologies, I assume households have full information about the technology and are not behavioral. To focus on household welfare, I do not consider general equilibrium effects or producer surplus. However, by reducing the estimation of consumer surplus from financial technologies to the estimation of demand, I provide theoretical grounding for the application of the broad array of models and methods that build on demand in static models.

I next estimate consumer surplus from financial technologies in three randomized control trials (Duflo et al., 2006; Karlan & Zinman, 2018, 2019). Each experiment introduced variation in prices: for deposits into a retirement savings account, for deposits into a commitment savings account, and for microfinance loans, respectively. As in the stylized example, the experimental treatment either reduced savers’ deposits conditional on their end-of-period balance, or increased borrowers’ loan amount conditional on their repayment schedule; in turn, this increased deposits and loan amount. I then estimate price elasticities of deposits and loan amount ranging from 4-to-14 across the three experiments; these are simply appropriate rescalings of estimated impacts from each paper. I then construct bounds on compensated price elasticities of deposits and loan amount: in general, these bounds are narrow compared to statistical uncertainty. Across the three experiments, the compensated price elasticities of deposits and loan amount correspond to annual consumer surplus of $7-to-$27 per $100 of deposits or loan amount.

I conclude by discussing the interpretation of my estimates of consumer surplus. I compare my estimates to existing experimental and quasi-experimental studies of the impacts of access to savings and credit technologies on household consumption, a common proxy for welfare; standard errors are large relative to my estimates, suggesting these studies are ex-post underpowered to detect impacts equal to my estimates of consumer surplus. De-
spite this, my estimates imply large welfare gains from financial technologies when scaled by
takeup of household credit and savings in low income countries.

**Related literature** This paper estimates the household welfare gains from financial tech-
nologies using a sufficient statistics approach, complementing estimates applying reduced
form and structural approaches. A growing literature has applied experimental and quasi-
experimental methods to estimate the household impacts of access to microfinance (e.g., Pitt
& Khandker, 1998; Karlan & Zinman, 2011; Kaboski & Townsend, 2012; Banerjee et al.,
2015b; Meager, 2019; Lane, 2020); this work has identified consumption smoothing, durable
consumption, and productive investment motives for borrowing, and has found mostly posi-
tive but imprecise impacts on proxies for household welfare. However, theory is necessary to
quantify household welfare: the sign of impacts of financial technologies on common welfare
proxies, including household consumption, need not be consistent with impacts on welfare,
such as when borrowing complements lumpy investments (Banerjee et al., 2015b). Kaboski
& Townsend (2011) estimate household welfare in a structural model of microfinance bor-
rowing that allows consumption smoothing, productive investment, and strategic default in
response to microfinance; I show my point estimates of consumer surplus from microfinance
are comparable to theirs. While reduced form and structural approaches shed light on and
quantify the mechanisms underlying borrower impacts of microfinance, the sufficient statis-
tics approach I employ complements these approaches by enabling estimation of consumer
surplus under a general neoclassical household model.

To produce estimates of household welfare gains from microfinance, this paper extends
textbook calculation of consumer surplus to models with incomplete financial markets. In
this regard, it is closest to Farhi et al. (2022), who provide formulas for consumer surplus
from small price changes with incomplete financial markets, and in addition produce a more
comprehensive price theory; in contrast, I focus only on consumer surplus, and as a result I
am able to derive results for large price changes and with more general preferences, non-linear
technology, and endogenous choice sets. I build most directly on results on exact consumer
surplus in a static model (Willig, 1976; Hausman, 1981). In particular, I represent exact
consumer surplus as the solution to a differential equation. In contrast to the static model,
multiple distinct solutions exist; for a given path of price changes, I select the solution
that does not require estimation of marginal utilities. I complement existing approaches
that estimate consumer surplus under fully parametrized structural models with incomplete
financial markets; Kaboski & Townsend (2011) estimates equivalent variation to improved
access to credit as an initial period transfer, while Lagakos et al. (2018) estimates dynamic
compensating consumption transfers for seasonal migration. I do so by proposing sufficient
statistics for equivalent transfers in models with incomplete financial markets; these sufficient statistics can be estimated from experimental variation in prices and income.

These results contribute to the analysis of welfare gains in dynamic household models with incomplete financial markets. I show that dynamically compensated demand elasticities are directly informative of welfare gains from general intertemporal substitution technologies, including financial technologies. For studies that isolate experimental or quasi-experimental variation in the price of financial technologies (Duflo et al., 2006; Karlan & Zinman, 2008, 2009, 2018, 2019; Alan & Loranth, 2013; Karlan et al., 2014; Ponce et al., 2017; Banerjee et al., 2018; Cai et al., 2020; Benetton, 2021; Benetton et al., 2021; Robles-Garcia, 2022), this enables transparent welfare analysis, as in Berkouwer & Dean (2022) and DeFusco et al. (2022). For studies that estimate welfare gains from financial technologies leveraging estimates of structural dynamic household models (Kaboski & Townsend, 2011), it suggests model implied demand elasticities should be assessed to evaluate the plausibility of welfare counterfactuals, as in Bryan et al. (2014), Lagakos et al. (2018), and Allcott et al. (2022).

Outline This paper is organized as follows. In Section 2, I present a household model with incomplete financial markets, in which I establish that consumer surplus is equal to the area under dynamically compensated demand. In Section 3, I provide practical guidance to researchers on estimating consumer surplus from financial technologies. In Section 4, I estimate uncompensated demand and bound compensated demand for financial technologies in three randomized control trials, which I apply to calculate consumer surplus. Section 5 concludes.

2 Consumer surplus with incomplete financial markets

In this section, I define and characterize consumer surplus with incomplete financial markets. In Section 2.1, I present a model of households facing incomplete financial markets across period-states, with general technologies, preferences, and choice sets. In Section 2.2, I derive Roy’s identity on the welfare impacts of small price changes. In Section 2.3, I apply Roy’s identity to calculate period-state-contingent equivalent variation from price changes, and approximations thereof. In Section 2.4, I discuss extensions featuring unobserved heterogeneity across households and unobserved shocks, and ranking welfare changes under incomplete financial markets.
2.1 Model

2.1.1 Environment

A utility maximizing household faces a menu of technologies for substitution across period-states $s \in \mathcal{S} \equiv \{1, \ldots, S\}$. The household makes an investment choice $b_s \in \mathbb{R}^N_+$ in each period-state $s$, for which it faces prices $p \in \mathbb{R}^N$; I let $b \equiv (b_1, \ldots, b_S)$ denote the full vector of investment choices, and I assume investment choices satisfy $b \in \mathcal{B} \subset \mathbb{R}^{SN}_+$. In each period-state $s$, the household holds wealth $y_s$, plus its period-state production as a function of investment choices $f_s(b)$. The household selects consumption $c_s \in \mathbb{R}$, subject to the period-state budget constraint: consumption $c_s$ plus investment expenditures $p \cdot b_s$ is weakly less than wealth $y_s$ plus production $f_s(b)$.

The household solves

$$V(p, y) = \max_{c,b} U(c, b)$$

subject to

$$c_s + p \cdot b_s \leq y_s + f_s(b), \quad \forall s \in \mathcal{S}$$

$$b \in \mathcal{B}$$

This setup treats time and uncertainty in a general manner. The set of period-states $\mathcal{S}$ enumerates both time and all potential histories of discrete shocks to the household. Transition probabilities are flexibly embodied in a utility function $U$, period-state production technology $f_s$, and investment choice set $\mathcal{B}$ that depend flexibly on the full vector of consumption choices $c$ and investment choices $b$. While this level of generality is unlikely to be necessary in applications, the resulting notation is more parsimonious than treating time and uncertainty separately, and it underscores which assumptions are necessary for the theoretical results in Sections 2.2 and 2.3. Note that this flexibility also implies that any choice variables with the subscript $s$ may therefore depend on the full history of shocks; I discuss implications of this for the empirical analysis of consumer surplus in Section 2.4, as conditioning on full histories of shocks may not be empirically plausible.

**Price $p$** For all results below on consumer surplus from price changes, it is crucial that prices $p$ enter the household problem only through the period-state budget constraint. Specifically, prices and wealth are perfect substitutes holding fixed investment choices. Importantly, changes in prices do not directly affect the investment choice set $\mathcal{B}$.

**Additional assumptions** I make the following additional assumptions, each of which I discuss below:

$$7$$
• Technology $f_s$ is continuous $\forall s \in S$.
• The investment choice set $\mathcal{B}$ is compact.
• Utility $U$ is continuous, and its derivatives with respect to consumption $\nabla_c U$ are continuous and strictly positive.

Technology $f_s(b)$ Production may depend flexibly on multiple previous period-states’ investment choices. Continuous technology over a compact choice set allows for lumpy investment technologies, and complementarities across mixed discrete and continuous investments.

For instance, a choice to default in period-state $s$ would contemporaneously manifest as increased output $f_s(b)$ (i.e., less repayment). However, it may affect future constraints on the household’s subsequent investment choices through the investment choice set $\mathcal{B}$, and output $f_{s'}(b)$ in future period-states $s'$ both positively (e.g., through reduced subsequent repayment) and negatively (e.g., through seized assets). Simultaneously, past investments in soil fertility on the households’ agricultural land may shape agricultural productivity, and may complement the application of modern agricultural inputs, as multiple past investment choices may interact to determine state-dependent output $f_s(b)$.

Investment choice set $\mathcal{B}$ That the full vector of investment choices $b$ lies in a compact set $\mathcal{B}$ provides substantial flexibility. It allows for discrete and continuous choice, and allows for broad dynamic incentives; it nests the case where each period-state’s investment choice set is continuous in previous choices and compact-valued, that is $b_s \in \mathcal{B}_s(b_1, \ldots, b_{s-1})$.

For instance, a household’s access to credit, and its credit limit, may depend on its credit history including any missed payments or default.

That investment choices are positive is without loss of generality, as an additional investment choice can be added with negative prices to capture negative investment choices, as in the case of saving and borrowing at a constant interest rate.

Utility $U(c, b)$ Utility need not be separable in consumption across period-states and in investment choices. This allows for deviations from both exponential discounting and expected utility maximization, as in Epstein & Zin (1989), and non-pecuniary investment motives (e.g., social stigma against default). That consumption choices $c$ are unrestricted within $\mathbb{R}^S$, and that utility is strictly increasing in consumption, are not without loss of generality – in particular, they imply that the household does not consume at the edge of its consumption choice set (i.e., the household does not consume at a subsistence constraint) and that each period-state’s budget constraint binds with equality, which I make use of and discuss in Section 2.2.
2.1.2 Existence of model solutions

The household problem in Equation 1 has consumption and investment solutions. Intuitively, as household utility is strictly increasing in consumption, the budget constraint binds in each period, and the household problem reduces to an investment choice problem. As technology is continuous in investments, the investment choice problem involves maximizing a continuous function on a compact set $B$, which is guaranteed to have a solution.

**Proposition 1.** A solution exists to the household problem in Equation 1.

I denote the consumption and investment solutions to the household problem in Equation 1 as $(C(p, y), B(p, y)) : \mathbb{R}^N \times \mathbb{R}^S \to 2^{\mathbb{R}^S} \times 2^B$. I also let $C_s(p, y)$ and $B_s(p, y)$ denote period-state $s$ consumption and investment solutions. By construction, utility $U(c, b)$ evaluated at these solutions is equal to indirect utility $V(p, y)$.

$$U(c, b) = V(p, y) \text{ if and only if } (c, b) \in (C(p, y), B(p, y))$$  \hfill (2)

2.1.3 Graphical representation of the model

In a version of the model with just two period-states and in which investment choices do not enter directly into utility, the model admits a simple graphical representation, which I present in Figure 2a. The household faces a consumption possibilities frontier, that is the set of feasible consumption choices as a function of prices, wealth, technology, and the investment choice set. With incomplete financial markets, the consumption possibilities frontier is nonlinear; this is a result of constraints in the investment choice set (e.g., the household can save but not borrow) and nonlinear technology (e.g., decreasing returns to scale). The household chooses its utility maximizing consumption bundle within its consumption possibilities frontier, which occurs at the tangency of the household’s indifference curves to its consumption possibilities frontier; this optimal consumption bundle is feasible at the household’s optimal investment bundle.

In Section 2.2, we will consider the impact of price changes on household welfare; changes in prices affect household utility by shifting the consumption possibilities frontier. This intuition suggests an application of the envelope theorem – the first order impact of a change in prices on household welfare is equal to the impact of the shift in the consumption possibilities frontier at the household’s optimal consumption bundle.
Figure 2: An example of equivalent variation with incomplete financial markets

(a) Optimal consumption at prices $p^0$ and $p^1$: $C(p^0, y), C(p^1, y)$

(b) Equivalent variation to change in prices from $p^0$ to $p^1$: $EV', EV'' \in EV(p^0, p^1; y)$
2.2 Roy’s identity

Although the model in Section 2.1 flexibly captures a broad range of technologies, preferences, and choice sets, it restricts that prices enter the household problem only through investment expenditures. As in Auclert (2019) and Farhi et al. (2022), this permits a generalization of Roy’s identity to the setting with incomplete financial markets – the impact of a small change in the price of an investment on utility is proportional to the sum of the marginal utility of period-state wealth times period-state investment.

**Theorem 1.** Indirect utility \( V(p, y) \) is absolutely continuous. In addition, suppose that the investment choice \( B(p, y) \) is a singleton; then the marginal utility of wealth is equal to the marginal utility of consumption

\[
\nabla_y V(p, y) = \nabla_c U(C(p, y), B(p, y))
\]

and the impact of a small change in prices on utility is equal to its impact holding fixed investment choices.

\[
\nabla_p V(p, y) = - \sum_{s=1}^S \frac{dV(p, y)}{dy_s} B_s(p, y)
\]

**Proof.** I consider the following investment choice problem equivalent to the household problem in Equation 1, derived from substituting for consumption with the binding budget constraints in Equation 1.

\[
V(p, y) = \max_{b \in B} U \left( y_s + f_s(b), p \cdot b_s \right)
\]

I then apply an envelope theorem (Corollary 4 of Milgrom & Segal, 2002) to Equation 5; the following immediately hold. First, \( V(p, y) \) is absolutely continuous. Second, if \( B(p, y) \) is a singleton, then \( \nabla_p V(p, y) = - \sum_{s=1}^S B_s(p, y) \frac{dU(C(p, y), B(p, y))}{dc_s} \) and \( \nabla_y V(p, y) = \nabla_c U(C(p, y), B(p, y)) \) (Equation 3). Substituting yields Equation 4.

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1As mentioned in Section 2.1.1, the assumption that utility is well defined over any real values of consumption is consequential – transforming the household problem in Equation 1 to the investment choice problem in Equation 5 relies on this assumption. Absent this assumption, the investment choice problem is defined only for investment choices that result in feasible (e.g., positive) consumption, and that set of investment choices might be affected by the price of investments. In that case, the choice set for the investment choice problem would be affected by prices, invalidating the application of the envelope theorem. However, this is only a concern if households select consumption at the boundary of their consumption choice set; in this case, a small change in prices could cause a discrete investment that has large impacts on utility to result in infeasible consumption, causing households that can no longer feasibly make that investment to experience a large change in utility. Note this is not a concern under common restrictions (e.g., Inada conditions) on utility that guarantee households do not consume at the boundary of their consumption choice set; these conditions may be met even if households face a subsistence constraint (e.g., Stone-Geary preferences).

2Details of the application of the envelope theorem are in Appendix A.
2.3 Consumer surplus

2.3.1 Equivalent variation

In the static household model, or models with complete financial markets, it is natural to define consumer surplus in units of permanent income; in contrast, with incomplete financial markets, permanent income need not exist. If a household cannot transfer income across period-states, it may not be possible to compensate a household for a change in prices in one period-state exclusively with transfers to another period-state. As an alternative, Farhi et al. (2022) consider Slutzky compensation, that is providing transfers that compensate for small price changes in each period-state.

I extend consumer surplus to models with incomplete financial markets by defining it as a vector of period-state-contingent transfers that holds utility fixed following a price change. I consider a path of price changes \( p^k : [0,1] \rightarrow \mathbb{R}^N \), and I implicitly define the equivalent variation for a change in prices from \( p^k \) to \( p^1 \) as

\[
V(p^k, y + EV(p^k, p^1; y)) = V(p^1, y)
\]  

where \( EV(p^k, p^1; y) : \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}^S \rightarrow 2^{\mathbb{R}^S} \). By construction, the household is indifferent between prices changing from \( p^k \) to \( p^1 \), and receiving the associated equivalent variation as period-state-contingent transfers. In contrast to the static household model, equivalent variation has multiple solutions, as there are many potential vectors of period-state-contingent transfers across which the household is indifferent.

I present a graphical representation of equivalent variation with incomplete financial markets in Figure 2b, in a version of the model with just two period-states and in which investment choices do not enter directly into utility. Changes in prices change the shape of the household’s consumption possibilities frontier, which affects the household’s optimal consumption bundle and its indirect utility. Similarly, income transfers shift the household’s consumption possibilities frontier; equivalent variation is the set of shifts of the consumption possibilities frontier which enable the household to achieve the same level of utility as from the change in prices. Note that equivalent variation shifts the consumption possibilities frontier to a tangency with the household’s indifference curve after the price change. As both indifference curves and the consumption possibilities frontier are nonlinear, different shifts to the consumption possibilities frontier will result in different points of tangency; in contrast to a model with complete financial markets, the choice of period-states in which to compensate the household is consequential.

To characterize equivalent variation, I consider solutions to the following differential equa-
tion derived from the definition of equivalent variation in Equation 6. I posit the existence of a differentiable solution to Equation 6, \( EV^*(p^k, p^1; y) \in EV(p^k, p^1; y) \), which I construct below in Equation 9. Differentiating Equation 6 with respect to \( k \), that is along the path of price changes, and substituting Roy’s identity (Equation 4) yields

\[
\sum_{s=1}^{S} \frac{dV(p^k, y + EV^*(p^k, p^1; y))}{dy_s} \left( \frac{dEV^*_s(p^k, p^1; y)}{dt} - B_s(p^k, y + EV^*(p^k, p^1; y)) \cdot \frac{dp^k}{dk} \right) = 0
\]

(7)

In general, solutions to Equation 7 may require knowledge of the full vector of marginal utilities of income. When markets are complete, asset prices determine relative marginal utilities of income; in other cases, estimating marginal utilities of income may require estimation of preferences, technology, and beliefs.

Note that a solution to Equation 7 is specific to the path of price changes. However, as solutions to this system solve Equation 6, they are exact measures of consumer surplus (Hausman, 1981).

I focus on a particularly tractable solution to Equation 7 that does not require knowledge of marginal utilities, by solving the system of differential equations

\[
\frac{dEV^*_s(p^k, p^1; y)}{dk} - B_s(p^k, y + EV^*(p^k, p^1; y)) \cdot \frac{dp^k}{dk} = 0
\]

(8)

This is simply the term inside the sum in Equation 7, and states that the change in period-state equivalent variation from a small price change is equal to the negative of the direct effect of the price change on compensated expenditures.

There are at least two reasons to normatively prefer solutions to Equation 8 over other solutions to equivalent variation. First, when demand is separable across period-states, that is \( B_s(p, y) \) is only affected by income in period-state \( s \), then static equivalent variation in each period-state solves Equation 8. Second, solutions to Equation 8 are suggested naturally by Roy’s identity (Equation 4): equivalent variation for small price changes maintains the feasibility of consumption choices holding fixed investment choices.

I solve Equation 8 for equivalent variation by integrating over the path of price changes

\[3\] I impose the initial condition that equivalent variation is 0 when prices do not change, that is that \( EV_s(p^1, p^1; y) = 0 \) for all \( s \in S \).

\[4\] Relatedly, Blackorby et al. (1984) and Keen (1990) demonstrate that static equivalent variation in a model with complete financial markets is equivalent variation only if there are no across period-state income effects.
from the initial price $p^1$.

$$EV^*_s(p^k, p^1; y) \equiv - \int_0^1 B_s(p^k, y + EV^*(p^k, p^1; y)) \cdot \frac{dp^k}{dk} dk$$

(9)

This solution is the integral over compensated demand with respect to price changes. It differs from static equivalent variation in that demand has across-period-state income effects.

**Theorem 2.** $EV^*(p^k, p^1; y) \in EV(p^k, p^1; y)$

2.3.2 Marshallian surplus

In the static household model, it is common to use Marshallian surplus, that is the integral over demand with respect to a change in prices, as a measure of consumer surplus when income effects are small (Willig, 1976; Vives, 1987). With incomplete financial markets, just as in the static household model, Marshallian surplus is exact when there are no income effects, as uncompensated and compensated demand are identical.

**Corollary 1.** Suppose there are no income effects, so $B(p^k, y) = B(p^k, y')$ for all $k \in [0, 1]$, $y, y' \in \mathbb{R}^S$. Define Marshallian surplus in period-state $s$ as

$$MS_s(p^k, p^1; y) \equiv - \int_0^1 B_s(p^k, y) \cdot \frac{dp^k}{dk} dk$$

(10)

Then $MS(p^k, p^1; y) \in EV(p^k, p^1; y)$.

2.3.3 Uncompensated and compensated demand elasticities

In practice, log linear approximations of uncompensated and compensated demand are commonly used, in part because they yield tractable approximations of Marshallian surplus and equivalent variation.\(^5\) For this purpose, and consistent with the empirical application in Section 4, I focus on changes in a single price in this section; for convenience, I use $B_s$ to refer to demand for the single investment choice for which prices are changing.\(^6\)

\(^5\)A log linear approximation of a household’s investment demand is unlikely to be appropriate, as investment choices are often discrete or discontinuously shaped by other discrete choices. However, a log linear approximation of average investment demand across households may be reasonable even when each household makes a discrete investment choice. I discuss the extension of the results in this section to allow for heterogeneous households in Section 2.4.

\(^6\)I extend these results to the case with multiple price changes in Online Appendix B; demand for each investment choice is affected by the full vector of price changes, and income effects are generated by all price changes.
I define the uncompensated price elasticity of demand

\[ \epsilon_s - 1 \equiv \frac{p^1}{B_s(p^1, y)} \frac{dB_s(p^1, y)}{dp^1} \]  

(11)

The term \( \epsilon_s \) can be interpreted as the price elasticity of expenditures on the investment choice; this corresponds to empirical estimates of impacts on deposits and loan amount in Section 4.

The corresponding compensated price elasticity of demand is

\[ \phi_s + \epsilon_s - 1 \equiv \frac{p^1}{B_s(p^1, y)} \frac{dB_s(p^1, y + EV^*(p^k, p^1; y))}{dp^k} \bigg|_{k=1} \]  

(12)

The difference between uncompensated and compensated price elasticities of demand is \(-\phi_s\), the income effect of the price change. Expanding the derivative of compensated demand in Equation 12 yields

\[ \phi_s = \sum_{s'=1}^S dp^1 \frac{B_s(p^1, y)}{dy_{s'}} \frac{B_{s'}(p^1, y)}{B_s(p^1, y)} \equiv MPBY_{s,s'} \]  

(13)

The negative of the income effect is simply the inner product of the household’s marginal propensity to make investment expenditures from period-state income MPBY\(_s\) \(\in \mathbb{R}^S\), and the growth rate of investment expenditures \(G_s \in \mathbb{R}^S\). In contrast to the static household model, income effects are both within and across period-states. In Section 4, I estimate \(G_s\) directly, and consider approaches to bound MPBY\(_s\) using the marginal propensity to consume.

These elasticities can be used to approximate Marshallian surplus and equivalent variation. If uncompensated and compensated demand are in fact log linear, then

\[ \frac{MS_s(p^0, p^1; y)}{p^1 B_s(p^1, y)} = 1 - \frac{(p^0/p^1)^{\epsilon_s}}{\epsilon_s} \]  

(14)

\[ \frac{EV_s(p^0, p^1; y)}{p^1 B_s(p^1, y)} = 1 - \frac{(p^0/p^1)^{\epsilon_s + \phi_s}}{\epsilon_s + \phi_s} \]  

(15)

These approximations depend only on the prices before and after the price change, and estimates of the price elasticity of investment choice expenditures \(\epsilon_s\) and income effects \(\phi_s\). I estimate the price elasticity of investment choice expenditures \(\epsilon_s\) and bound income effects \(\phi_s\) across three empirical applications in Section 4.
2.4 Extensions

The model and results above considered a single household, and did not normatively interpret consumer surplus.

In Online Appendix C, I follow Hausman & Newey (2016) and consider bounds on average consumer surplus under unobserved heterogeneity across households and shocks; I construct these bounds by substituting bounds on the matrix of average intertemporal marginal propensities to spend into Equation 12.7

In Online Appendix D, I consider the application of consumer surplus to ranking policies, focusing on the case with unobserved heterogeneity across households and states of the world. I discuss ranking policies using the net present value (NPV) of average consumer surplus, aggregated using the social planner’s cost of transferring income across periods and states. I note it retains a similar interpretation, and limitations, to the NPV of average consumer surplus in the static model – it implicitly assumes the contrasted policies benefit the same groups of households conditional on fixed characteristics (Finkelstein & Hendren, 2020) or income (Hendren, 2020), a strong assumption in the presence of unobserved heterogeneity in preferences across households (Sallee, 2022).

3 Practical guidance on consumer surplus from financial technologies

In Section 2, I established that consumer surplus from a financial technology is equal to the area under dynamically compensated demand; I this section, I lay out key considerations for applied researchers estimating consumer surplus from financial technologies. In Section 3.1, I discuss the definition of prices, and therefore quantities, used to construct demand. In Section 3.2, I describe assumptions, and associated tests, on dynamically compensated demand, that permit the use of statically compensated or uncompensated demand. In Section 3.3, I consider common extensions and potential sources of bias in the estimation of consumer surplus from financial technologies, presented with comparisons to the static case.

7I allow unobserved heterogeneity in two steps. First, in Online Appendix C.1, I allow for unobserved heterogeneity across households. Second, in Online Appendix C.2, I allow for unobserved heterogeneity across states of the world, such as unobserved aggregate or household shocks; I make the additional assumption that household intertemporal marginal propensities to spend do not systematically differ out of income received across histories of shocks with respect to compensated demand.
3.1 Defining, estimating, and graphing demand for financial technologies

I break down the estimation of consumer surplus from a financial technology, or consumer surplus more generally, into three steps: defining demand, estimating demand, and graphically representing surplus as the area under demand. I describe deviations from the static model at each step for the model with incomplete financial markets.

**Defining demand**  In Section 2, demand for a financial technology was defined as period-state specific quantities purchased as a function of prices and income. In the static model for traditional consumer goods, it is typically clear how to define quantities, prices, and income. For financial technologies, this is less obvious. However, just as in the static model, the essential ingredients are that prices and income are perfect substitutes holding fixed quantity, and that price times quantity equals expenditures. I discuss three examples below, and I present three additional examples in the applications in Section 4.

- In Section 1, I presented an example of a static model of demand for a loan. Price is negative loan amount per $1200 of total monthly payments, quantity is total monthly payments divided by $1200, expenditures are negative loan amount, and income is temporary income in the period in which the loan is taken.
- Benetton (2021) present a static model of supply and demand for mortgages; on the demand-side, the borrower chooses one mortgage product from an offered menu. These mortgages have a period during which interest rates are fixed, after which they float with market rates; for tractability, suppose that households pay interest, but not principal, during the fixed rate period. In this case, prices are annual interest rates, quantities are initial loan balances, expenditures are annual loan payments during the fixed rate period, and income is expected annual income during the fixed rate period.
- Benetton et al. (2021) and Robles-Garcia (2022) expand on the static model of supply and demand in Benetton (2021); mortgage products also carry an origination fee. In this case, an alternative definition of prices are origination fees (applied in Robles-Garcia (2022)), quantities are binary take-up (with loan balance and interest rates treated as attributes that differentiate mortgage products), expenditures are the origination fee if the household takes up the loan, and income is temporary income the period in which the mortgage product is selected.

**Estimating demand**  As in the static model, estimating demand for a financial technology principally involves isolating exogenous variation in its supply; each of the applications in
Section 4 leverages a randomized control trial that introduces random variation in consumer prices. The source of random variation in prices determines the interpretation of consumer surplus for the financial technology.

When price changes are temporary and of known duration, consumer surplus is equivalent variation for access to the technology for the duration of the price change; when the duration of the price change is relatively short, a static model is appropriate. In Sections 4.1 and 4.2, I present applications where price changes were temporary and I apply a static model to the estimation of consumer surplus.

When price changes are permanent, consumer surplus is equivalent variation for access to the technology from the time of the price change. The dynamics of demand with respect to permanent price changes are essential to the interpretation of consumer surplus from permanent price changes. In Section 4.3, I present an application in which the price elasticity of demand is increasing over time, and I account for these dynamics when estimating and interpreting consumer surplus.

Compensated demand for financial technologies depends on intertemporal income effects. In Section 3.2, I discuss assumptions under which intertemporal income effects can be ignored, and income effects can be handled as in the static model. In three applications in Section 4, I consider approaches to bound intertemporal income effects in order to bound compensated demand relative to uncompensated demand.

Graphing demand and consumer surplus Consumer surplus from financial technologies can be represented as the area under compensated demand. In Section 4, I argue that uncompensated demand is a reasonable approximation of compensated demand for many financial technologies, and I discuss approaches to justifying the assumption that uncompensated demand is a reasonable approximation of compensated demand in Section 3.2. When variation in prices comes from a randomized control trial, as in Section 4, estimating uncompensated demand is simply fitting a curve through average quantities at each randomized price.

In Figure 3, I present estimates of uncompensated demand from each application in Section 4. In each application, the estimates of Marshallian surplus from treatment in Tables 1 and 2 correspond to the integral of demand over a change in prices; the difference between Marshallian surplus and equivalent variation is attributable to income effects.

3.2 Simplifying assumptions on dynamically compensated demand

In this paper, I develop and demonstrate an approach to estimating consumer surplus from financial technologies, building on tools for the estimation of consumer surplus in the static
Figure 3: Demand and Marshallian surplus from treatment

(a) Retirement savings (Duflo et al., 2006)

(b) Commitment savings (Karlan & Zinman, 2018)

(c) Microfinance (Karlan & Zinman, 2019)

Notes: Experimental estimates of demand for savings and microfinance from Tables 1 and 2 are presented in this figure. Points are sample means of demand conditional on treatment assignment in Panels a and b, and exponentiated sample means of log demand conditional on treatment assignment in Panel c, with error bars indicating 95% confidence intervals.
model. The approach builds on three key sufficient statistics for consumer surplus from financial technologies, defined in Section 2.3.3. The first is the vector of price elasticities $\epsilon$: I discuss defining, estimating, and visualizing $\epsilon$ in Section 3.1. The second is the growth in demand for the financial technology $G$. The third is the matrix of marginal propensities to spend on the financial technology in period-state $s$ from temporary income in period-state $s'$, MPBY. Estimating MPBY is particularly challenging, especially for future temporary income shocks; instead, I propose approaches to constructing plausible bounds on MPBY in the applications in Section 4. These latter two sufficient statistics $G$ and MPBY are used to construct the income effects from dynamic price changes, as one period’s demand may be affected by income in other period-state. It is only through these generalized income effects that estimation of consumer surplus in a dynamic model differs from the estimation of consumer surplus in the static model.

As an alternative to bounding intertemporal marginal propensities to spend MPBY, two approaches are common in literature estimating household welfare gains from a financial technology. The first approach implicitly assumes that there are no intertemporal income effects; that is, that past and anticipated income shocks do not affect use of the financial technology, or that the financial technology is only used once. In this case, consumer surplus from a financial technology is the area under statically compensated demand. The second approach implicitly assumes that there are no income effects, that is, income shocks do not affect use of the financial technology. In this case, consumer surplus from a financial technology is the area under uncompensated demand.8

No intertemporal income effects As seen in Equation 13, two conditions are required for intertemporal income effects to arise: households must make the investment choice in multiple periods, and income in other periods must affect demand. When households make the investment choice in a single period, or when income in other periods does not affect demand, then a static model of demand is appropriate.

Households do not make an investment choice in multiple periods for many financial technologies. Benetton et al. (2021) consider the case of mortgages by first-time home buyers, which by construction are only made once. As a consequence, the static model they employ is appropriate for calculating consumer surplus from mortgages for first-time home buyers.9 In addition, their consumer surplus calculations are likely robust to parametric assumptions, 8As discussed in Section 3.1, the duration of price changes, and dynamic responses of demand during the price change, remain important to the interpretation of consumer surplus under these simplifying assumptions.

9If households can change the timing of their first-time home purchase in response to changes in prices, this approach implicitly sums undiscounted consumer surplus within household across periods. However, this is a general limitation of applying a static framework to the estimation of consumer surplus.
conditional on their estimated demand matching the appropriate empirical price elasticity of demand.

Household income in other periods may not affect demand for many financial technologies. For example, when investments are made infrequently in contexts with severe credit constraints, income effects are unlikely to affect demand in other periods. Berkouwer & Dean (2022) evaluate consumer surplus from improved cookstoves, which are replaced approximately every two years, and find that credit constraints starkly limit willingness-to-pay. In this environment, a transfer equal to consumer surplus from the cookstove is unlikely to affect demand for the cookstove two years before or after.

No income effects In other cases, income effects may not be first order; as demonstrated in Corollary 1, in this case Marshallian surplus (the area under uncompensated demand) is equal to consumer surplus.

As discussed in Section 2.3.2, and seen in Equation 12 and 13, income effects are small when the investment choice represents a small share of income, and a plausible marginal propensity to spend is therefore small, the uncompensated price elasticity of demand is large. Karlan et al. (2014) study demand for index insurance in Ghana – they estimate a price elasticity of demand of -2, while the value of index insurance purchased does not exceed 0.05 as a share of annual agricultural income. These values differ by an order of magnitude, suggesting Marshallian surplus closely approximates consumer surplus from index insurance.

Alternatively, when intertemporal income effects can be ignored and the investment choice is discrete, as in Berkouwer & Dean (2022) discussed above, Marshallian surplus is equal to compensating variation for a price decrease (Bhattacharya, 2015).

3.3 Extensions

The application of consumer surplus as a measure of welfare gains from financial inclusion rests on the strong assumptions laid out in Section 2. Many of these assumptions are also often applied in the static model; a large literature has relaxed many of these assumptions, which can be similarly applied to the incomplete financial markets setting.

- **Information** Households are often not aware of the price schedules they face for financial technologies; Duflo et al. (2007) present evidence that complexity of incentives for retirement savings in the United States may limit takeup, while Karlan & Zinman (2019) argue that their finding that price elasticities of demand for microfinance in Mexico are increasing in time since an interest rate decrease may reflect slow diffusion of information about the decrease. In static settings, estimates of demand under full
information can be used to recover consumer surplus under incomplete information (Chetty et al., 2009; Allcott & Taubinsky, 2015); my theoretical results suggest these approaches could be extended to the estimation of consumer surplus under incomplete information with incomplete financial markets.

- **Behavioral biases** Households may make investment choices subject to behavioral biases; in the case of microfinance, Banerjee (2013) and Zinman (2014) present a number of models which generate such biases, including present-focused preferences.\(^{10}\) Mullainathan et al. (2012), Farhi & Gabaix (2020), and Lerva (2022) provide examples of how the estimation of consumer surplus can be augmented with additional sufficient statistics to account for internalities and externalities. These methods build on estimates of compensated demand, such as those I produce in this paper, and permit many comparative statics on welfare to be represented graphically using both observed and “de-biased” demand. In recent work, Allcott et al. (2022) estimate consumer surplus from payday lending in the US, building on their finding that borrowers are sophisticated but present-focused; the intuition above suggests that their welfare analysis is likely robust to many of their modeling assumptions, conditional on implied demand and their estimates of internalities.

- **Producer surplus and supply** Both consumer surplus and producer surplus are necessary for a full accounting of the welfare gains from financial inclusion. However, estimating producer surplus in this setting is complicated by the presence of selection and moral hazard (Zinman, 2014). Recent work has directly estimated the impacts on microfinance lender profits of price changes or the introduction of new products (Karlan & Zinman, 2019; Lane, 2020; DeFusco et al., 2022). Impacts of price changes on profits can be used to construct average and marginal cost curves, accounting for adverse selection and moral hazard, and to characterize the market equilibrium when combined with estimates of demand, leveraging tools applied to the analysis of markets for health insurance Einav et al. (2010), as in DeFusco et al. (2022). With additional instruments, more flexible estimation of supply is possible (Benetton, 2021; Benetton et al., 2021; Robles-Garcia, 2022). My theoretical results in Section 2 clarify that the current practice of abstracting from households’ complex multidimensional decision making environment, and instead focusing on matching empirical demand for the financial technology, does not bias estimates of consumer surplus from price changes, as in the static case.

- **General equilibrium** An immediate consequence of incomplete financial markets is

\(^{10}\) An analogous concern is that the preferences of a paternalistic social planner may differ from the preferences of the household.
that households vary in their marginal returns to investment, and reallocation across households in general equilibrium may therefore have first order impacts on efficiency, in addition to the usual distributional consequences. Buera et al. (2021) estimate a model of microfinance in general equilibrium, in which entry and exit of and reallocation across heterogeneous entrepreneurs are triggered by microfinance and its impacts on equilibrium wages, while Breza & Kinnan (2021) provide evidence that equilibrium impacts of microfinance on wages are large. As a complementary approach, recent work has decomposed the general equilibrium impacts of microeconomic shocks on aggregate welfare into a partial equilibrium effect (which I estimate), and an allocative efficiency effect (Baqaee & Farhi, 2020; Bau & Matray, 2020; Sraer & Thesmar, 2020); under these approaches, the impacts of financial inclusion on allocative efficiency depend crucially on its impacts on reallocation towards high marginal return entrepreneurs.

4 Consumer surplus from savings and microfinance

In this section, I apply the framework in Section 2, and approach discussed in Section 3.1, to estimate consumer surplus from savings and microfinance. I build on three randomized control trials (Duflo et al., 2006; Karlan & Zinman, 2018, 2019) which introduce experimental variation in interest rates for retirement savings accounts in the United States (Section 4.1), commitment savings accounts in Philippines (Section 4.2), and microfinance loans in Mexico (Section 4.3), respectively.

4.1 Retirement savings (Duflo et al., 2006)

4.1.1 Context and experimental design

I leverage experimental variation in retirement savings match rates studied by Duflo et al. (2006) to estimate the saver surplus from retirement savings. The experiment was implemented with H&R Block, a large tax preparer in the United States, for a single retirement savings product, the X-IRA.

Retirement savings product I briefly summarize the description of the retirement savings product from Duflo et al. (2006), and incorporate it into the model in Section 2. At the time households enrolled in the experiment prepare their taxes with H&R Block, they are offered the opportunity to make a one-time deposit into an X-IRA with their forthcoming tax refund, with a minimum deposit of 300 USD. The deposit into the X-IRA may or may not be tax deductible, and households may or may not face tax penalties for withdrawing
from their X-IRA before age 59.5. Lastly, households enrolled in the experiment are offered a match rate \( r \) on deposits up to 1000 USD; in practice, 85% of deposits in treatment arms with a match occurred below or at the 1000 USD threshold. The experiment, including the match offer, was implemented during a single tax preparation season in 2005.

I therefore represent the retirement savings product in the household problem in Equation 1 as

\[
\frac{1}{1+r} b_1 \in B_1 \equiv \{0\} \cup [300, 1000]
\]

where \( \frac{1}{1+r} b_1 \) and \( b_1 \) are the household’s deposit into the retirement savings account, excluding and including the match, respectively.

**Experimental variation in match rates** Households enrolled in the experiment were randomized into a 0%, 20%, and 50% match rate, revealed at the time of the offer to deposit into the savings account. I let \( D \in \{0, 1, 2\} \) correspond to a household’s treatment assignment in order of increasing match rate. I normalize \( p^0 = 1 \) to be the price of deposits under 0% match rate, yielding prices of deposits under a 20% and 50% match rate of \( p^1 = \frac{1}{1+0.2} = 0.833 \) and \( p^2 = 0.667 \), respectively.

**Prices and investment choice set** One limitation of this modeling approach for this experiment is that prices enter the household problem both through the budget constraint and the investment choice set. Note that we can rewrite the household’s investment choice set as

\[
b_1 \in B_1 \equiv \{0\} \cup \left[\frac{300}{p^D}, \frac{1000}{p^D}\right]
\]

In this case, changes in prices now have an additional welfare impact through shifting the households’ choice set of deposits including the match (that is, \( b_1 \)). However, note that households can always choose to deposit more than 1000 USD in the X-IRA, albeit with no match; this suggests a revealed preference approach to place an upper bound on how much households value the impact of price decreases on their investment choice set, which could be implemented with microdata from the experiment.

**Additional modeling considerations** Key aspects of the tax incentives and withdrawal penalties around the savings product above complicate modeling, yet can be flexibly accommodated in the model in Section 2. Not only is the timing of withdrawals from the savings product endogenous to deposits, but tax deductions and penalties will vary both
with deposits and also other behavioral responses endogenous to deposits. I can model tax deductions, withdrawals, and tax penalties flexibly through \( f_a(b) \), which allows these outcomes to depend not only on the amount the household deposited and its previous withdrawals, but also on any other investment choices made by the household that affect its tax status.

4.1.2 Demand elasticities and consumer surplus

**Estimation strategy** I leverage the experimental variation in match rates to estimate the price elasticity of demand for commitment savings. Let \( m_i \) denote observed deposits by household \( i \) into the offered X-IRA retirement savings account, and let \( D_i \in \{0, 1, 2\} \) indicate the treatment assignment for household \( i \). I estimate

\[
\log \mathbb{E}_i[m_i|D_i] = \alpha + \epsilon \log p^{D_i}
\] (16)

This fits a log linear model of average deposits as a function of prices through estimates of average deposits conditional on treatment assignment from Duflo et al. (2006); absent microdata from the experiment, I estimate this model by generalized method of moments.

**Price elasticity of deposits** \( \epsilon \) I present my estimate of the price elasticity of deposits for retirement savings in Table 1, along with associated estimates of compensated demand and consumer surplus from commitment savings. As deposits correspond to expenditures in the model in Section 2, my estimate corresponds to \( \epsilon \) (rather than the price elasticity of demand \( \epsilon - 1 \)) in the log linearization of demand in Section 2.3.3. I estimate \( \epsilon = -4.2 \), with a standard error of 0.3; the large variation in match rates introduced by the experiment results in a highly precise estimate of the price elasticity of deposits.

**Bounding income effects \( \phi \) and the compensated deposits elasticity** As the experiment consists of a one-shot offer, I model the match rate as lasting for a single period, and the income effect term in the compensated elasticity of deposits \( \phi \) simplifies to \( \phi = \text{MPBY} \), the marginal propensity to deposit into the retirement savings account from temporary income. Using a one year MPC of 0.5, and allowing for the possibility that households have access to multiple savings accounts, I upper bound \( \text{MPBY} \leq 1 - \text{MPC} = 0.5 \). I further assume that households would not decrease their retirement savings deposits in response to a temporary positive income shock, implying \( \text{MPBY} \geq 0 \). This yields \( \phi \in [0, 0.5] \), and in turn yields bounds on the compensated price elasticity of deposits \( \phi + \epsilon \in [-4.2, -3.7] \).
Table 1: Price elasticities of savings demand and saver surplus

<table>
<thead>
<tr>
<th></th>
<th>(Duflo et al., 2006)</th>
<th>(Karlan &amp; Zinman, 2018)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
</tbody>
</table>

**Panel A: Regression estimates of demand elasticities**

<table>
<thead>
<tr>
<th>log deposit amount</th>
<th>(USD)</th>
<th>(PHP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>3.37</td>
<td>5.64</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.08)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>log price (ε)</td>
<td>-4.22</td>
<td>-9.74</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(8.16)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.233]</td>
</tr>
</tbody>
</table>

# of household observations

|                      | 13,962 | 6,696 |

**Panel B: Estimates of compensated demand elasticities and consumer surplus**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensated deposit elasticity (φ + ϵ) (Lower bound)</td>
<td>-4.22</td>
<td>-9.74</td>
</tr>
<tr>
<td>Compensated deposit elasticity (φ + ϵ) (Upper bound)</td>
<td>-3.72</td>
<td>-9.24</td>
</tr>
</tbody>
</table>

**Surplus from high interest rate treatment per $100 control group deposit amount**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marshallian surplus</td>
<td>$107.43</td>
<td>$1.58</td>
</tr>
<tr>
<td>(USD)</td>
<td>($7.29)</td>
<td>($0.10)</td>
</tr>
<tr>
<td>Equivalent variation (Lower bound)</td>
<td>$94.58</td>
<td>$1.57</td>
</tr>
<tr>
<td>Equivalent variation (Upper bound)</td>
<td>$107.43</td>
<td>$1.58</td>
</tr>
</tbody>
</table>

**Surplus from savings per $100 deposit amount**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marshallian surplus</td>
<td>$23.70</td>
<td>$10.27</td>
</tr>
<tr>
<td>(USD)</td>
<td>($1.48)</td>
<td>($8.61)</td>
</tr>
<tr>
<td>Equivalent variation (Lower bound)</td>
<td>$23.70</td>
<td>$10.27</td>
</tr>
<tr>
<td>Equivalent variation (Upper bound)</td>
<td>$26.89</td>
<td>$10.83</td>
</tr>
</tbody>
</table>

Notes: Regression coefficients and consumer surplus are presented in this table, with standard errors in parentheses and p-values in brackets. In Panel A, regression coefficients are presented; Column 1 is estimated by GMM matching average outcomes conditional on randomly assigned prices reported in Duflo et al. (2006), while Column 2 is estimated by Poisson PML with heteroskedasticity-robust standard errors. In Panel B, bounds on compensated deposit elasticities and consumer surplus are presented.
Marshallian surplus from and equivalent variation bounds for high interest rate treatment and commitment savings

I apply Equations 14 and 15 for Marshallian surplus and equivalent variation, respectively, to calculate the welfare gains from the high match rate treatment and from retirement savings.

The high match rate treatment, expressed per $100 of control group deposits, generated Marshallian saver surplus of $107.4, with a standard error of $7.3. As the high match rate treatment generated a large increase in deposits through a large decrease in prices, its associated surplus is closely tied to the estimated demand elasticity.

Marshallian saver surplus from retirement savings is simply $-\frac{1}{\epsilon}$, implying that retirement savings generates saver surplus of $23.7 per $100 of deposits, with a standard error of $1.5. Bounds on equivalent variation are of similar width to the 95% confidence interval for the estimate of Marshallian saver surplus. Although estimates of the demand elasticity for retirement savings are the most precise that I analyze in this paper, due to the large sample and large variation in match rates, statistical uncertainty in the point estimate remains similar in magnitude to bias in Marshallian saver surplus as a measure of equivalent variation due to income effects.

4.2 Commitment savings (Karlan & Zinman, 2018)

4.2.1 Context and experimental design

I leverage experimental variation in interest rates on commitment savings accounts studied by Karlan & Zinman (2018) to estimate the saver surplus from commitment savings. The experiment was implemented with First Valley Bank in the Philippines, for a commitment savings product, Gihandom Savings.

Commitment savings product

I briefly summarize the description of the commitment savings product from Karlan & Zinman (2018), and incorporate it into the model in Section 2. Households were recruited for the experiment in a door-to-door marketing campaign, and randomized into three commitment savings account offers, two of which we focus on which vary the annual interest rate $r$: one with $r = 1.5\%$, and the other with $r = 3.0\%$. Following the offer, households could open the account from the bank with a minimum deposit of 100 PHP; households would set a goal amount of at least 2,000 PHP, and a goal term of between 3 months and 2 years, and could withdraw only after both the goal term and amount were met.

I therefore represent the commitment savings product in the household problem in Equa-
tion 1 as

\[
\frac{1}{1+r} b_t \in B_t(b_{t-1}) = \begin{cases} 
  t \geq \text{Goal date and } b_{t-1} \geq \text{Goal amount} & [0, \infty) \\
  t < \text{Goal date or } b_{t-1} < \text{Goal amount} & [b_{t-1}, \infty)
\end{cases}
\]

where \( \frac{1}{1+r} b_t \) and \( b_t \) are the household’s deposits and end-of-period balance, respectively, in the commitment savings account, and \( r \) is the annual interest rate. I model the deposit decision as annual, to align the model with the outcome variable I use in the analysis in Section 4.2.2 (deposits during the 12 months following the introduction of the commitment savings account).

**Experimental variation in interest rates** Households were randomized at the time of the marketing visit into a 1.5% and 3% offered interest rate. I let \( D \in \{0, 1\} \) correspond to the household’s treatment assignment in order of increasing interest rate. I normalize \( p^0 = 1 \) to be the price of deposits under the 1.5% interest rate, yielding the price under a 3% interest rate of \( p^1 = \frac{1+0.015}{1+0.030} = 0.985 \).

**Prices and investment choice set** One limitation of this modeling approach for this experiment relates to the interaction of interest rates and the commitment aspect of the savings accounts leading to changes in the investment choice set. When a household receives additional interest, its balance increases, and as a result the household is committed to maintaining a higher balance until they reach their goal, corresponding to a higher lower end of the interval \( B_t(b_{t-1}) \). This tightening of the investment choice set would reduce my estimates of the welfare gains from commitment savings in Section 4.2.2.

**Additional modeling considerations** Key aspects of the commitment savings product above and the household’s deposit and withdrawal decisions complicate modeling; despite this complexity, these can be flexibly accommodated in the model in Section 2. First, households may withdraw early in the event of hardship. Borrowers’ investment choice set \( B_t(b_{t-1}) \) may be period-state-specific, permitting withdrawals in response to shocks. In addition, if early withdrawal comes with some ordeal costs or punishment, the early withdrawal decision may directly enter into utility. Second, households face a minimum initial deposit; this may be modeled similarly to the minimum initial deposit for retirement savings in Section 4.1.

### 4.2.2 Demand elasticities and consumer surplus

**Estimation strategy** I leverage the experimental variation in interest rates to estimate the price elasticity of demand for commitment savings. Let \( m_t \) denote observed deposits
by household $i$ during the 12 months following the introduction of the commitment savings account, and let $D_i \in \{0, 1\}$ indicate the treatment assignment for household $i$. I estimate

$$\log \mathbb{E}_i[m_i|D_i] = \alpha + \epsilon \log p^{D_i}$$

(17)

This is similar to the estimation of impacts in Karlan & Zinman (2018), with three differences. First, I transform treatment assignment into log prices, with control group log price normalized to 0 ($\log p^0 = 0$); this rescales the treatment effect of high interest rates on deposits by the inverse of the treatment group log price, that is $1/\log p^1 \approx -67$. Second, I do not include any control variables, as including controls does not meaningfully affect the results; the intercept $\alpha$ therefore equals the log of average deposits at control group interest rates.

**Price elasticity of deposits $\epsilon$** I present my estimate of the price elasticity of deposits for commitment savings in Table 1, along with associated estimates of compensated demand and consumer surplus from commitment savings. As deposits correspond to expenditures in the model in Section 2, my estimate corresponds to $\epsilon$ (rather than the price elasticity of demand $\epsilon - 1$) in the log linearization of demand in Section 2.3.3. I estimate $\epsilon = -9.7$, with a standard error of 8.2; while the point estimate is not significantly different from 0, it is sufficiently large to rule out much larger price elasticities of deposits, which would correspond to much smaller consumer surplus from savings.

**Bounding income effects $\phi$ and the compensated deposits elasticity** Absent data on future years of commitment savings deposits, I model the change in interest rates as lasting for just one year. In this case, the income effect term in the compensated elasticity of deposits $\phi$ simplifies to $\phi = \text{MPBY}$, the marginal propensity to deposit into the commitment savings account from temporary income. Using a one year MPC of 0.5, and allowing for the possibility that households have access to multiple savings accounts, I upper bound $\text{MPBY} \leq 1 - \text{MPC} = 0.5$. I further assume that households would not decrease their commitment savings deposits in response to a temporary positive income shock, implying $\text{MPBY} \geq 0$. This yields $\phi \in [0, 0.5]$, and in turn yields bounds on the compensated price elasticity of deposits $\phi + \epsilon \in [-9.7, -9.2]$.

**Marshallian surplus from and equivalent variation bounds for high interest rate treatment and commitment savings** I apply Equations 14 and 15 for Marshallian surplus and equivalent variation, respectively, to calculate the welfare gains from the high interest rate treatment and from commitment savings.
The high interest rate treatment, expressed per $100 of control group deposits, generated Marshallian saver surplus of $1.6, with a standard error of $0.1. This estimate is much more precise than the estimated demand elasticity – the interest rate variation induced by the experiment is sufficiently small that the impacts of the high interest rate treatment on saver welfare are well approximated by the reduced deposits savers can make holding fixed their end-of-year balance. Closely related to this point, bounds on equivalent variation to the high interest rate treatment are not meaningfully different from Marshallian surplus.

For larger price changes, the estimated elasticity plays a much larger role; Marshallian saver surplus is simply $-1/\epsilon$, implying that commitment saving generates saver surplus of $10.3 per $100 of deposits, with a standard error of $8.6$. Relatedly, bounds on equivalent variation are also wider, at $[10.3, 10.8]$; however, the width of these bounds is small relative to statistical uncertainty in the point estimate for Marshallian saver surplus.

4.3 Microfinance (Karlan & Zinman, 2019)

4.3.1 Context and experimental design

I leverage experimental variation in microfinance interest rates studied by Karlan & Zinman (2019) to estimate the borrower surplus from microfinance. The experiment was implemented with Mexico’s largest microfinance lender, Compartamos Banco, for a single microfinance lending product, Crédito Mujer.

Microfinance product I briefly summarize the description of the microfinance credit product from Karlan & Zinman (2019), and incorporate it into the model in Section 2. Each period, borrowing groups of women are evaluated based on observed characteristics, including general creditworthiness and past borrowing and repayment behavior with the microfinance lender. Borrowing groups are offered the opportunity to take a non-collateralized loan up to a threshold based on these characteristics; offered thresholds range from 4,000 MXN to 24,000 MXN. Borrowers are obligated to repay the loan in identical weekly installments over 16 weeks, which are calculated based on three factors: the initial principal, value-added tax on interest payments, and “add-on” interest calculated based on the initial principal rather than the balance of the loan. Monthly add-on interest before the experiment ranged from 4.0% to 5.0%, with the interest determined geographically and on the basis of past borrowing and repayment behavior. I let $b_{t-1} \in \mathbb{R}_+^N$ denote past borrowing and repayment behavior, $r_t(b_{t-1}) : \mathbb{R}_+^N \rightarrow \{0.040, 0.045, 0.050\}$ denote the pre-experimental interest rate, $\bar{b}_t(b_{t-1}) : \mathbb{R}_+^N \rightarrow [4000, 24000]$ denote the offered threshold, and $b_{t,1}$ denote the principal on new loans taken in period $t$. 

30
A borrower who takes a loan of $b_{t,1} \in \mathbb{R}_+$ is obligated to repay \( \frac{1 + 1.15 \times 4 \times r_t(b_{t-1})}{16} b_{t,1} \) in each of the 16 weeks following period \( t \), which includes 15% value-added tax on the 4 months of repayments of monthly add-on interest. I represent the lending product in the household problem in Equation 1 as

\[
b_{t,1} \in \mathcal{B}_{t,1}(b_{t-1}) \equiv [0, \bar{b}_t(b_{t-1})] \\
f_t(b) = -\sum_{\tau=1}^{16} \frac{1 + 1.15 \times 4 \times r_{t-\tau}(b_{t-\tau-1})}{16} b_{t-\tau,1}
\]

**Additional modeling considerations** Key aspects of the lending product above and the household’s borrowing decisions complicate modeling; crucially, these can be flexibly accommodated in the model in Section 2. First, default and delayed repayment do occur (1% and 10% of loans, respectively), so households repayments are also a function of their past and current decisions to delay repayment or default. I can capture these responses by incorporating these delay and default decisions into the vector of investment choices \( b_t \), and by allowing flexible technology \( f_t(b_t) \). Second, there may be additional extensive margin borrowing costs, such as group formation, and default costs, such as social pressure from other group members. I can represent these costs by allowing borrowing and default decisions in \( b_t \) to enter directly into utility \( U(c, b) \). Third, borrowers are ostensibly required to make deposits into a personal savings account proportional to their borrowing, but these requirements were partially enforced. These partially enforced requirements may generate period-state-specific interactions across the borrower’s investment choices in the borrower’s choice set \( \mathcal{B}_t(b_{t-1}) \), or alternatively in the borrower’s utility function \( U(c, b) \).

**Experimental variation in interest rates** The experiment was implemented in the context of the microfinance lender decreasing its monthly add-on interest rates. At the start of the experiment, the microfinance lender’s geographic regions had their monthly add-on interest rates decreased across all borrowers; this decrease was 0.5pp for randomly assigned control regions, and 1pp for randomly assigned treatment regions. Let \( D \) be an indicator for treatment assignment; following the start of the experiment, weekly repayment obligations (as a fraction of the initial principal) were decreased to \( \frac{1 + 1.15 \times 4 \times (r_t(b_{t-1}) - (1 + D) \times 0.005)}{16} \). This decrease is well approximated by a constant proportion: repayment obligations fell by an additional 1.91% to 1.98% (1.95% average) in treatment regions relative to control regions. The experiment was implemented in 78 regions and lasted for 29 months from 2007 through 2009, after which the lender decreased control region interest rates to the level in treatment regions.
I therefore model the experiment as permanently shifting the price $p$ of the microfinance lending product; prices are negative as households borrow, with more negative prices (price decreases) corresponding to larger principal holding fixed repayment obligations. I normalize $p^0 = -1$ to be the price in the control group, and I calculate low interest rate treatment group prices $p^1 = -\frac{1}{1-0.0199} = -1.020$. Similarly, I let $b_{t,1}(p^D)$ represent microfinance borrowing under assigned prices $p^D$. The monthly loan amount I observe in the data under treatment assignment $D$ are therefore equal to $-p^D b_{t,1}(p^D)$; microfinance borrowing choices $b_{t,1}(p^D)$ are proportional to repayment obligations, while prices $p^D$ shift principal holding fixed repayment obligations.

This modeling choice implicitly imposes additional assumptions. First, it treats the price change as the same across counterfactual add-on interest rates, while in fact price changes differ slightly; this error is an order of magnitude smaller than other sources of estimation error, and I therefore ignore it. Second, it imposes that the investment choice set $B_t(b_{t-1})$ is not affected by prices; that is, it imposes that the maximum borrowing threshold is proportional to repayment obligations, and therefore $b_{t,1}$, rather than loan amount $-p^D b_{t,1}$. When maximum borrowing thresholds are on loan amount $-p^D b_{t,1}$, price decreases also reduce welfare by tightening the constraint on microfinance borrowing choices $b_{t,1}$; at the extreme, a constrained household that does not value the future does not benefit from such a price decrease, as they do not value their reduced repayment obligations. In practice, Karlan & Zinman (2019) find that only 19% of loans are at the maximum borrowing threshold, suggesting that bias due to overestimation of the welfare gains from price decreases to constrained borrowers is likely to be small.

### 4.3.2 Demand elasticities and consumer surplus

**Estimation strategy** I leverage the experimental variation in interest rates to estimate the price elasticity of demand for microfinance. Let $\ell_{g,n}$ denote total observed loan amount in region $g$ in month $n$, and let $t(n) \in \{1, 2, 3\}$ correspond to the year of the experiment. I let $\bar{\ell}_{g,0}$ be the pre-experimental loan amount in region $g$. Further, I let $D_g \in \{0, 1\}$ indicate the treatment assignment for region $g$. I estimate

$$\log \ell_{g,n} = \alpha_{t(n)} + \epsilon_{t(n)} \log(-p^D_g) + \gamma_{t(n)} \log \bar{\ell}_{g,0} + \zeta_{g,n}$$

(18)

This is similar to the estimation of impacts in Karlan & Zinman (2019), with two differences. First, I transform treatment assignment into log prices, with control group log price normalized to 0 ($\log(-p^0) = 0$); this rescales the treatment effect of low interest rates on log loan amount by the inverse of the treatment group log price, that is $1/\log(-p^1) \approx 50$. Second,
I control for log total loan amount from the two pre-experimental months for which data is available, log $\ell_{g,0}$, to improve precision (McKenzie, 2012). In addition, I demean baseline values of the dependent variable $\ell_{g,0}$; the intercept $\alpha_{t(n)}$ therefore equals average log loan amount at control group interest rates in Year $t$.

**Price elasticity of loan amount** $\epsilon$ I present my estimates of the price elasticity of loan amount for the microfinance lending product in Table 2, along with associated estimates of compensated demand and consumer surplus from the microfinance lending product. As loan amount corresponds to negative expenditures in the model in Section 2, my estimate corresponds to $\epsilon$ (rather than the price elasticity of demand $\epsilon - 1$) in the log linearization of demand in Section 2.3.3. I estimate the price elasticity of demand grows over time, with $\epsilon_1 = 6.4$ with a standard error of 3.2 in Year 1 and growing to $\epsilon_3 = 14.1$ with a standard error of 5.3 in Year 3. These estimates are consistent with those in Karlan & Zinman (2019), albeit with a different scale.

**Bounding income effects $\phi$ and the compensated loan amount elasticity** In contrast to Sections 4.1 and 4.2, I estimate dynamic $\epsilon_t$. To produce bounds on income effects, I therefore consider bounds on both $-\text{MPBY}_{t,t'}$, the marginal propensity to take loan amount in period $t$ from temporary income in period $t'$ (negative because loan amount corresponds to negative expenditures), and on $G_{t,t'}$, the growth rate of loan amount.

First, I bound $-\text{MPBY}_{t,t'}$. I do so in two steps – I make an assumption on intertemporal marginal propensity to consumes, and I then bound the marginal propensity to take loan amount relative to the marginal propensity to consume. In the first step, I assume that the marginal propensity to consume in period $t$ from shocks to temporary income in period $t'$ is $\text{MPC}_{t,t'} = \text{MPC}(1 - \text{MPC})|t-t'|$. I argue this functional form is a reasonable starting point. For shocks to future income ($t' > t$), it is the unique functional form that yields a proportionally declining marginal propensity to consume from future income, and also a marginal propensity to consume from permanent income of 1 (Gelman et al., 2016; Ralston et al., 2017). For shocks to past income ($t' < t$), it imposes that households consume a constant fraction $\text{MPC}$ of the temporary income shock in each period, and save the remainder. In the second step, I construct lower and upper bounds in the following manner. For an upper bound, I assume that households borrow no more than their marginal propensity to consume from temporary income, that is $-\text{MPBY}_{t,t'} \leq \text{MPC}_{t,t'}$. For a lower bound for shocks to future income, I assume that households will not decrease borrowing in response to a positive shock to future income, that is $-\text{MPBY}_{t,t'} \geq 0$ when $t' > t$. For a lower bound for shocks to past or current income, I assume that borrowing will not decrease
Table 2: Price elasticities of microfinance demand and borrower surplus

(Karlan & Zinman, 2019)

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

**Panel A: Regression estimates of demand elasticities**

<table>
<thead>
<tr>
<th></th>
<th>log loan amount ('000 MXN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>8.61 (0.04) [0.000]</td>
</tr>
<tr>
<td></td>
<td>8.95 (0.05) [0.000]</td>
</tr>
<tr>
<td></td>
<td>9.16 (0.06) [0.000]</td>
</tr>
<tr>
<td>log loan amount, Year 0, demeaned</td>
<td>0.72 (0.04) [0.000]</td>
</tr>
<tr>
<td></td>
<td>0.53 (0.04) [0.000]</td>
</tr>
<tr>
<td></td>
<td>0.43 (0.05) [0.000]</td>
</tr>
<tr>
<td>log price ($\epsilon_t$)</td>
<td>6.40 (3.22) [0.047]</td>
</tr>
<tr>
<td></td>
<td>10.50 (4.30) [0.015]</td>
</tr>
<tr>
<td></td>
<td>14.08 (5.25) [0.007]</td>
</tr>
<tr>
<td># of regions</td>
<td>78</td>
</tr>
<tr>
<td># of region-month observations</td>
<td>936</td>
</tr>
</tbody>
</table>

**Panel B: Estimates of compensated demand elasticities and consumer surplus**

<table>
<thead>
<tr>
<th></th>
<th>Compensated loan amount elasticity ($\phi_t + \epsilon_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Lower bound)</td>
</tr>
<tr>
<td>Compensated loan amount elasticity ($\phi_t + \epsilon_t$)</td>
<td>4.40 8.25 11.70</td>
</tr>
<tr>
<td></td>
<td>(Upper bound)</td>
</tr>
<tr>
<td>Compensated loan amount elasticity ($\phi_t + \epsilon_t$)</td>
<td>6.90 11.25 14.95</td>
</tr>
</tbody>
</table>

**Surplus from low interest rate treatment per $100 control group loan amount**

<table>
<thead>
<tr>
<th></th>
<th>Marshallian surplus</th>
<th>Equivalent variation (Lower bound)</th>
<th>Equivalent variation (Upper bound)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$2.11 ($0.07)</td>
<td>$2.07 ($0.07)</td>
<td>$2.12 ($0.12)</td>
</tr>
<tr>
<td></td>
<td>$2.20 ($0.10)</td>
<td>$2.15 ($0.10)</td>
<td>$2.22 ($0.12)</td>
</tr>
<tr>
<td></td>
<td>$2.28 ($0.12)</td>
<td>$2.23 ($0.12)</td>
<td>$2.30 ($0.12)</td>
</tr>
</tbody>
</table>

**Surplus from microfinance per $100 loan amount**

<table>
<thead>
<tr>
<th></th>
<th>Marshallian surplus</th>
<th>Equivalent variation (Lower bound)</th>
<th>Equivalent variation (Upper bound)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$15.62 ($7.84)</td>
<td>$14.48 ($7.84)</td>
<td>$15.21 ($7.84)</td>
</tr>
<tr>
<td></td>
<td>$9.53 ($3.90)</td>
<td>$8.89 ($3.90)</td>
<td>$9.28 ($3.90)</td>
</tr>
<tr>
<td></td>
<td>$7.10 ($2.65)</td>
<td>$6.69 ($2.65)</td>
<td>$7.05 ($2.65)</td>
</tr>
</tbody>
</table>

**Notes:** Regression coefficients and consumer surplus are presented in this table, with standard errors in parentheses and p-values in brackets. In Panel A, regression coefficients are presented; Columns 1 through 3 are estimated by OLS with robust standard errors clustered at the region-level. In Panel B, bounds on compensated loan amount elasticities and consumer surplus are reported.
by more than the household’s marginal propensity to save from temporary income, that is
\[-\text{MPBY}_{t,t'} \geq -\frac{1 - \text{MPC}_{t,t'}}{\text{MPC}}\] when \(t' \leq t\). In summary, this yields
\[
\text{MPBY}_{t,t'} \in \begin{cases} 
-\text{MPC}(1 - \text{MPC})^{t-t'}, (1 - \text{MPC})^{1+t-t'} & t' \leq t \\
-\text{MPC}(1 - \text{MPC})^{t'-t}, 0 & t' > t
\end{cases}
\]

Second, I bound \(G_{t,t'} \in [1, 1.5]\), with 1.5 corresponding to the ratio of Year 2-to-Year 1 loan amount in the low interest rate treatment group.

Using a one year MPC of 0.5, this yields
\[
\phi_t = \sum_{t' = 1}^{\infty} \text{MPBY}_{t,t'} G_{t,t'} \in [-2.5 + 0.5t, 1 - 0.5t]
\]

These bounds on income effects are much wider than the bounds on static income effects in Section 4.1 and 4.2 – this is because the price reduction results in income effects from past, current, and future periods. Despite this, these bounds remain relatively tight compared to the demand elasticities estimated in Table 2. For the estimated Year 1 price elasticity of loan amount of \(\epsilon_1 = 6.4\), the bounds on the compensated price elasticity of loan amount are \(\phi_1 + \epsilon_1 \in [4.4, 6.9]\), while for the estimated Year 3 price elasticity of loan amount of \(\epsilon_3 = 14.1\), the bounds on the compensated price elasticity of loan amount are \(\phi_3 + \epsilon_3 \in [11.7, 15.0]\). As in the previous two examples, statistical uncertainty is larger than potential bias from using uncompensated price elasticities as a measure of compensated price elasticities.

**Marshallian surplus from and equivalent variation bounds for low interest rate treatment and microfinance** I apply Equations 14 and 15 for Marshallian surplus and equivalent variation, respectively, to calculate the welfare gains from the low interest rate treatment and from the microfinance lending product.

The low interest rate treatment, expressed per $100 of control group loan amount, generated Marshallian borrower surplus of $2.1 in Year 1, with a standard error of $0.1, and $2.3 in Year 3, with a standard error of $0.1. This estimate is much more precise than the estimated demand elasticity – the interest rate variation induced by the experiment is sufficiently small that the impacts of the low interest rate treatment on borrower welfare are well approximated by the increased loan amount savers can receive holding fixed their repayments. Closely related to this point, bounds on equivalent variation to the low interest rate treatment are not meaningfully different from Marshallian surplus.

For larger price changes, the estimated elasticity plays a much larger role; Marshallian borrower surplus is simply \(1/\epsilon\), implying that the microfinance lending product generates borrower surplus of $15.6 per $100 of deposits in Year 1, with a standard error of $7.8, and
$7.1 per $100 of deposits in Year 3, with a standard error of $2.7. Relatedly, bounds on equivalent variation are also wider, at [$14.5, $22.7] in Year 1 and [$6.7, $8.6] in Year 3; however, the width of these bounds is small relative to statistical uncertainty in the point estimate for Marshallian borrower surplus.

4.4 Discussion of results

Interpretation of magnitude of surplus   Leveraging experimental estimates of demand for savings and microfinance, across a range of contexts, I estimate Marshallian surplus from investment technologies ranging from $7 per $100 of loan amount from microfinance in Mexico to $24 per $100 of deposits for retirement savings in the United States. One natural question in response to these estimates is whether we should think of them as small or large.

To interpret my estimates of Marshallian surplus, I begin by comparing them to comparable existing estimates of welfare gains from savings and microfinance. In Table 3, I present experimental and quasi-experimental estimates of the annual welfare gains from the introduction of new savings and microfinance technologies, expressed per $100 of loan amount or deposits. As most of the estimates are for microfinance, I focus my discussion on the comparison to and the interpretation of my estimates of Marshallian surplus from microfinance.

First, and most comparable to my estimates, Kaboski & Townsend (2011) estimate a structural model of savings, borrowing, and investment that closely matches quasi-experimental impacts of microfinance in Thailand; they use the estimated model to calculate equivalent variation from microfinance, albeit as a single initial period transfer. I use this estimate to construct a conservative lower bound on annual equivalent variation of $4 per $100 of loan amount, somewhat smaller than my estimates of Marshallian surplus from microfinance.

Second, I compare my estimates to a more common approach: using household consumption as a proxy for household welfare. The prevalence of this approach is despite the well-known limitation that impacts of savings or microfinance on household consumption may fail to recover the correct sign of welfare gains (Kaboski & Townsend, 2011; Banerjee et al., 2015b). In Table 3, I highlight another limitation: these estimates tend to be extremely imprecise; with two exceptions, the standard errors on the estimated impacts of microfinance or savings on household consumption are at least twice as large as my largest estimate of Marshallian surplus, suggesting even a meta-analysis across these estimates would be underpowered to reject that they are, on average, the same as my estimates of Marshallian surplus. The two exceptions inform future research using household consumption as a proxy for household welfare. The first exception, Attanasio et al. (2015), estimates impacts of mi-
Table 3: Comparing surplus from microfinance and savings to their impacts on consumption

<table>
<thead>
<tr>
<th>Source</th>
<th>Country</th>
<th>Outcome</th>
<th>Year</th>
<th>Estimate</th>
<th>(SE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dupas et al. (2018)</td>
<td>MWI</td>
<td>Household expenditures</td>
<td>2011</td>
<td>$129</td>
<td>($260)</td>
</tr>
<tr>
<td></td>
<td>UGA</td>
<td>Household expenditures</td>
<td>2011</td>
<td>$37</td>
<td>($164)</td>
</tr>
<tr>
<td>Source</td>
<td>Country</td>
<td>Outcome</td>
<td>Year</td>
<td>Estimate</td>
<td>(SE)</td>
</tr>
<tr>
<td>Surplus from savings</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Table 1</td>
<td>USA</td>
<td>Marshallian surplus</td>
<td>2005</td>
<td>$24</td>
<td>($1)</td>
</tr>
<tr>
<td>Table 1</td>
<td>PHL</td>
<td>Marshallian surplus</td>
<td>2007</td>
<td>$10</td>
<td>($8)</td>
</tr>
<tr>
<td>Consumption impacts of savings</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angelucci et al. (2015)</td>
<td>MEX</td>
<td>Food expenditures</td>
<td>2011</td>
<td>$66</td>
<td>($248)</td>
</tr>
<tr>
<td>Attanasio et al. (2015)</td>
<td>MNG</td>
<td>Household expenditures</td>
<td>2009</td>
<td>$25</td>
<td>($14)</td>
</tr>
<tr>
<td>Augsburg et al. (2015)</td>
<td>BIH</td>
<td>Household expenditures</td>
<td>2010</td>
<td>-$111</td>
<td>($56)</td>
</tr>
<tr>
<td>Banerjee et al. (2015a)</td>
<td>IND</td>
<td>Household expenditures</td>
<td>2008</td>
<td>$26</td>
<td>($95)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Household expenditures</td>
<td>2010</td>
<td>-$188</td>
<td>($198)</td>
</tr>
<tr>
<td>Crépon et al. (2015)</td>
<td>MAR</td>
<td>Household consumption</td>
<td>2009</td>
<td>-$68</td>
<td>($71)</td>
</tr>
<tr>
<td>Kaboski &amp; Townsend (2011)</td>
<td>THA</td>
<td>Household consumption</td>
<td>2002</td>
<td>$139</td>
<td>($39)</td>
</tr>
<tr>
<td>Surplus from microfinance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Table 2</td>
<td>MEX</td>
<td>Marshallian surplus</td>
<td>2007</td>
<td>$16</td>
<td>($8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Marshallian surplus</td>
<td>2009</td>
<td>$7</td>
<td>($3)</td>
</tr>
<tr>
<td>Kaboski &amp; Townsend (2011)</td>
<td>THA</td>
<td>Equivalent variation (Lower bound)</td>
<td>2002</td>
<td>$4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Equivalent variation, present value</td>
<td>2002</td>
<td>$69</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Experimental estimates of the impacts of savings and microfinance on consumption and estimates of saver and borrower surplus from microfinance, per $100 of deposits and loan amount, are reported in this table, with standard errors in parentheses. The construction of estimates from other papers is described in Online Appendix A.

crofinance in Mongolia on household expenditures of $25 (with a standard error of $14) per $100 loan amount, significantly different from $0 but not from my estimates of Marshallian surplus from microfinance in Mexico. Their estimate is relatively precise despite a small number of observations for two primary reasons: they have a strong first stage, with treated households 51pp more likely to take a loan from the MFI than control households, and the loans were large, with an average loan size of two months of household consumption. The second exception, Kaboski & Townsend (2011), estimate impacts of microfinance in Thailand on household consumption of $139 (with a standard error of $39) per $100 loan amount. As they also find these large positive impacts persist over time, this estimate on its own implies much larger welfare gains than their structural model; in Online Appendix E, I demonstrate that because the impact of microfinance on consumption does not discount uncertain returns to risky investments, it may be biased upwards as a measure of consumer surplus.

With this context, I then ask if we should think of the magnitudes of Marshallian surplus
from savings and microfinance as small or large. First, I note that my estimates of Marshallian surplus are not significantly different from estimates of the impacts of microcredit loans to women in Bangladesh on household consumption from Pitt & Khandker (1998) ($18 per $100 loan amount) and from Khandker (2005) ($15 and $21 per $100 loan amount); while the internal and external validity of these estimates has been widely debated, the magnitudes themselves were interpreted as implying that microfinance cost-effectively increases household consumption (Morduch, 1999). Second, I note that it is often the case that households’ borrowing and saving is large as a share of household consumption. Household credit as a share of GDP averages roughly 10% in low income countries where data is available, and rises to over 40% in high income countries (International Monetary Fund, 2006). In low income countries, my estimates of Marshallian surplus from microfinance in Table 2 would therefore correspond to consumer surplus from household credit of 1% - 2% of GDP. Lastly, I note that the price elasticities of demand I estimate for microfinance and savings are large relative to conventional estimates of demand for consumer goods. As consumer surplus is inversely proportional to the compensated price elasticity of demand, these large elasticities correspond to relatively small consumer surplus. However, as noted above, household borrowing and saving is large as a share of household consumption. The consumer surplus per dollar lent or saved may be small, and therefore often empirically elusive, but the intense use of financial technologies corresponds to large aggregate welfare gains from financial inclusion.

Interpretation of accuracy and precision of elasticities  To visualize the variation in my estimates of price elasticities of demand, I plot the absolute values of my estimates of the price elasticities of deposits and loan amounts $\epsilon$, along with bounds on compensated elasticities $\phi + \epsilon$, in Figure 4. With the exception of Duflo et al. (2006), statistical imprecision in the estimates is substantial, as they introduced an order of magnitude more variation in prices than Karlan & Zinman (2018) and Karlan & Zinman (2019). In all cases, bounds on compensated elasticities are narrower than 95% confidence intervals – improvements in the efficiency of estimates of consumer surplus from financial technologies are likely to come primarily from more precise estimates of price elasticities of demand, which will likely require the introduction of large variation in prices.

5 Conclusion

In this paper, I estimate the partial equilibrium consumer welfare gains from savings and credit. I show how experimental estimates of the impacts of changes in interest rates on savings and borrowing can be used to construct welfare-relevant price elasticities of demand.
I apply these price elasticities of demand to estimate consumer surplus from retirement savings in the United States, commitment savings in the Philippines, and microfinance in Mexico. These estimates can be interpreted as the value households place on access to savings and microfinance. Consumer valuations of savings and credit are policy relevant – they can be compared to the costs of subsidizing the expansion of savings and credit to new households.

Broadly, this paper contributes to the estimation of economic surplus with incomplete financial markets. It provides theoretical and empirical evidence that the primacy of demand elasticities for estimation of consumer surplus persists when financial markets are incomplete.

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A Model appendix

Proof of Theorem 1  To apply Corollary 4 of Milgrom & Segal (2002) in the proof of Theorem 1, I apply the following to the investment choice problem in Equation 5. First, I consider prices, wealth, and the objective function for the investment choice problem in Equation 5 in a neighborhood of prices $p^0$ and wealth $y^0$. Let $p^k = p + kdp$, $y^k = y + kdy$, and $\Upsilon(b, k) = U((y^k_s + f_s(b) - p^k \cdot b_s)_{s=1}^S, b)$. Note that $\Upsilon(b, 0) = V(p^0, y^0)$ for $b \in B(p, y)$. For each $k \in [-1, 1]$, fix $b(p^k, y^k) \in B(p^k, y^k)$. $\Upsilon(b, k)$ is continuous in $b$, and $\frac{\partial \Upsilon}{\partial k} = \sum_{s=1}^S (-b_s \cdot dp + dy_s) \frac{dU}{dc_s}$ is continuous in $(b, k)$, because derivatives of $U$ with respect to consumption are continuous. The assumptions for Corollary 4 on $X$, $X^*(t)$, and $f(x, t)$ then hold for $\mathcal{B}$, $B(p^k, y^k)$, and $\Upsilon(b, k)$, respectively (with $k$ substituting for $t$).
Online Appendix A  Construction of estimates in Table 3

Dupas et al. (2018)  Table 4, Column 1 reports impacts on formal savings balance of 8.780 (1.270) and 3.883 (0.605) in Uganda and Malawi, respectively. Table 5, Column 4 reports impacts on monthly expenditures of 0.273 (1.20) and 0.416 (0.84) in Uganda and Malawi, respectively. I calculate impacts in Uganda of \((12 \times 0.273) / 8.780 = 0.37\) with standard error of \((12 \times 1.20) / 8.780 = 1.64\). I calculate impacts in Malawi of \((12 \times 0.416) / 3.883 = 1.29\) with standard error of \((12 \times 0.84) / 3.883 = 2.60\).

Angelucci et al. (2015)  Table 2B reports impacts on loan amount from experimental lender of 629 (74) for the 3 most recent loans over the last 2 years, and Table 6 reports impacts on weekly food expenditures of 4 (15). I calculate impacts of \(4 \times 52 / (629 / 2) = 0.66\) and standard errors of \(15 \times 52 / (629 / 2) = 2.48\).

Attanasio et al. (2015)  Table 2, Panel B reports impacts on loan amount from experimental lender of 365932 (44233) for outstanding loans. Table A2 reports average maturity between 199 and 243 days (about 0.6 years). Table 6 reports impacts on log per capita consumption of 0.109 (0.061). Table 1 reports average annual household consumption of 2800000. I estimate total loan amount over the average maturity of the loan as double outstanding loans. I calculate impacts of \((2800000 \times 0.109) / (365932 \times 2 / 0.6) = 0.25\) with standard error of \((2800000 \times 0.061) / (365932 \times 2 / 0.6) = 0.14\).

Augsburg et al. (2015)  Table 2 footnotes reports impacts on loans from experimental lender of 1.1. Text reports average loan amount of 1,653, and households were surveyed 9 - 13 months after initial dispersal (about 0.9 years). Table 6 reports impacts on annual expenditures per capita of -647.9 (327.6), and Table 1 reports average household size of 3.45. I calculate impacts of \(3.45 \times -647.9 / (1653 \times 1.1 / 0.9) = -1.11\) with standard error of \(3.45 \times 327.6 / (1653 \times 1.1 / 0.9) = 0.56\).

Banerjee et al. (2015a)  Table 2, Panel B reports impacts on outstanding loan amount from experimental lender of 1,334 (230) at one year and 979 (287) at three years. Text reports average maturity of 50 weeks (about 1 year). Table 6 reports impacts on monthly expenditures per capita of 10.24 (37.22) at one year and -48.83 (51.53) at three years, and Table 1B reports average household size of 5.645 at one year and 6.269 at three years. I estimate total loan amount over the average maturity of the loan as double outstanding
loans. I calculate one year impacts of $10.24 \times 12 \times 5.645 / (1334 \times 2) = 0.26$ with standard errors of $37.22 \times 12 \times 5.645 / (1334 \times 2) = 0.95$ and three year impacts of $-48.83 \times 12 \times 6.269 / (979 \times 2) = -1.88$ with standard errors of $51.53 \times 12 \times 6.269 / (979 \times 2) = 1.98$.

**Crépon et al. (2015)** Table 2, Panel B reports impacts on outstanding loan amount, for loans over 12 months prior to survey, from experimental lender of 796 (103), with an average loan maturity of 16 months, which I interpret to capture total loan amount over the 12 months prior to survey. Table 6 reports impacts on monthly household consumption -45 (47). I calculate impacts of $-45 \times 12 / 796 = -0.68$ and standard errors of $47 \times 12 / 796 = 0.71$.

**Kaboski & Townsend (2011)** Table 5 reports impacts on consumption per unit of loan amount of 1.39 (0.39). Section 5.3 reports equivalent variation for a single period transfer of 7000 per 10100 loaned; I calculate equivalent variation for a single period transfer of $7000 / 10100 = 0.69$. I calculate a lower bound on flow equivalent variation, by multiplying this value by the estimated interest rate on savings from Kaboski & Townsend (2011), as the single period transfer must provide at least as much utility as a transfer equal to the flow of interest from permanently holding the 7000 transfer as savings. Kaboski & Townsend (2011) estimate an interest rate on savings of 0.054, I therefore calculate a lower bound on flow equivalent variation of $0.054 \times 0.69 = 0.04$.

**Online Appendix B  Compensated price elasticity of demand when multiple prices change**

Define $\tilde{dp} \equiv \frac{dp^k}{dk} \bigg|_{k=1}$ to be the initial gradient of price changes. Separately, define implicitly $\tilde{B}_s(p, y) \equiv B_s(p, y) \cdot \frac{dp}{dk} \bigg|_{k=1} = B_s(p, y) \cdot \tilde{dp}$. The compensated price elasticity of demand in Equation 12 can be generalized to the case where multiple prices are changing.

$$\phi_s + \epsilon_s - 1 \equiv \frac{1}{B_s(p^1, y)} \left. \frac{d\tilde{B}_s(p^k, y + EV^*(p^k, p^1, y))}{dk} \right|_{k=1}$$  \hspace{1cm} (S1)

When $B_s(p, y)$, and therefore $p$, is scalar, and $\tilde{dp} = p$, then Equation S1 collapses to Equation 12.
Define the associated generalization of the uncompensated price elasticity of demand
\[
\epsilon_s - 1 \equiv \frac{1}{\bar{B}_s(p_1, y)} \left. \frac{d\tilde{B}_s(p^k, y)}{dk} \right|_{k=1}
\] (S2)

Lastly, note that
\[
\phi_s = \sum_{s'=1}^{S} \frac{d\tilde{B}_s(p^1, y)}{dy_{s'}} \frac{\tilde{B}_s'(p^1, y)}{\tilde{B}_s(p^1, y)}
\] (S3)

Equation S3 is analogous to Equation 13; however, when multiple prices are changing, scalar investment demand is calculated using the gradient of price changes as weights.

**Online Appendix C  Bounds on average consumer surplus with unobserved heterogeneity**

**Online Appendix C.1  Unobserved heterogeneity across households**

The model in Section 2.1 implicitly assumed a representative household. In the benchmark static model, assuming a representative household is equivalent to constant income effects across households; with unobserved heterogeneity across households, consumer surplus is no longer point identified, but bounds on income effects can be used to construct bounds on average consumer surplus (Hausman & Newey, 2016).

I therefore closely follow the approach of Hausman & Newey (2016) to construct bounds on average consumer surplus with incomplete financial markets and unobserved heterogeneity across households. Let \( i \) index households, and \( E_i \) denote expectation with respect to the distribution of households \( i \). Allowing for unobserved heterogeneity across households, taking the expectation of Equation 9 yields
\[
E_i \left[ EV^*_s(p^k, p^1; y, i) \right] = \int_{k}^{1} E_i \left[ B_s(p^i, y + EV^*(p^k, p^1; y, i); i) \right] \cdot \frac{dp^i}{dk} dk
\] (S4)

The intuition for the nonidentification of average surplus with heterogeneous income effects is visible in Equation S4. Average consumer surplus is the integral over average compensated demand, but compensation is equal to household consumer surplus, rather than average consumer surplus. The correct income effect to calculate average compensated demand is therefore the compensated demand weighted average of income effects, which is not point
I now derive bounds on consumer surplus with unobserved heterogeneity across households. For notational convenience, as in Section 2.3.3, I assume a single price is changing, and use $B_s(p, y; i)$ to denote investment expenditures on the single good for which prices are changing.

I assume the following bounds hold on income effects for each household.

$$MPBY_s \cdot EV^*(p^k, p^1; y, i) \leq p^k \left( B_s(p^k, y + EV^*(p^k, p^1; y, i); i) - B_s(p^k, y; i) \right) \leq MPBY_s \cdot EV^*(p^k, p^1; y, i) \quad (S5)$$

With a single period-state, as in Hausman & Newey (2016), $MPBY$ and $\overline{MPBY}$ are scalar bounds on the marginal propensity to spend out of income. Equation S5 generalizes to the case with multiple period-states: bounds on income effects $MPBY_s$ and $\overline{MPBY}_s$ place upper and lower bounds on the marginal propensity to make investment expenditures in period-state $s$ from income in each period-state $s' \in S$.\textsuperscript{A1}

I use the bounds on income effects in Equation S5 to bound the compensated price elasticity of demand; this follows Equation 12, but allows for unobserved heterogeneity across households. Taking expectations of Equation S5 across households and differentiating yields

$$\sum_{s' = 1}^S MPBY_{s,s'} \frac{\mathbb{E}_i[B_{s'}(p^1, y; i)]}{\mathbb{E}_i[B_s(p^1, y; i)]} \leq \frac{p^1}{\mathbb{E}_i[B_s(p^1, y; i)]} \left( - \frac{d\mathbb{E}_i[B_s(p^1, y; i)]}{dp^1} \bigg|_{p^k = 1} \right) \leq \sum_{s' = 1}^S MPBY_{s,s'} \frac{\mathbb{E}_i[B_{s'}(p^1, y; i)]}{\mathbb{E}_i[B_s(p^1, y; i)]} \quad (S6)$$

At the lower and upper bound, Equation S6 is identical to the equation for the difference between the compensated and uncompensated price elasticities of demand in Equation 13, but with demand replaced by average demand. The approach to estimating consumer surplus in Section 2.3 is therefore robust to unobserved heterogeneity across households, with demand replaced by average demand and given bounds on income effects $MPBY_s$ and $\overline{MPBY}_s$; I discuss application-specific bounds in Section 4.

As noted by Hausman & Newey (2016), replacing average demand with, for example,\textsuperscript{A1} The bounds in Equation S5 effectively impose that household demand is Lipschitz continuous in income, which requires substantially stronger assumptions than those made in the general framework in Section 2.1. However, it is sufficient that these bounds hold in expectation across households, that is on consumer surplus weighted average income effects, to derive Equation S6.
average log demand, would not be consistent with Equation S6, something I avoid in the applications in Section 4.

Online Appendix C.2 Unobserved shocks

The model in Section 2.1 flexibly captures uncertainty by allowing nonseparable utility and technology across period-states \( s \in S \); however, it is implicit in the analysis in Section 2.3 that the period-state \( s \) is observed. Often, important shocks associated with the realized state are observed, such as the history of price and weather realizations (Rosenzweig & Udry, 2020). However, in practice many are unobserved. I therefore decompose period-state \( s = (t, h), s' = (t', h') \), where \( t, t' \in \{1, \ldots, T\} \) denotes time (and potentially observed state and history of states) while \( h, h' \in \{1, \ldots, H\} \) denotes the history of realized shocks.

I let \( \mathbb{E}_h[\cdot | \mathcal{i}] \) denote expectation with respect to the distribution of histories \( h \), conditional on household \( i \). With this notation, I replace household \( i \)'s period-state \( s \) consumer surplus in Equation S4 with its average period \( t \) surplus across potential histories of realized shocks.

\[
\mathbb{E}_i \left[ \mathbb{E}_h \left[ EV_{t,h}^*(p^k, p^1; y, i) | \mathcal{i} \right] \right] = \int \mathbb{E}_i \left[ \mathbb{E}_h \left[ B_{t,h}(p^k, y + EV^*_i(p^k, p^1; y, i); i) | \mathcal{i} \right] \right] \frac{dp^*}{d\kappa} d\kappa \quad (S7)
\]

Additionally allowing for unobserved shocks when calculating average consumer surplus in Equation S7 is similar to allowing for unobserved heterogeneity across households in Equation S4. In both cases, unobservables shape decisions, and the analysis in Section 2 averages across these unobservables.

However, unobserved heterogeneity within household is distinct in one key respect from unobserved heterogeneity across households. There are income effects across histories \( h \) within household \( i \), but there are not income effects across households \( i \) – this complicates the construction of reasonable bounds on income effects. I therefore consider the decomposition of bounds on income effects into two terms. The first term of the lower bound, \( \text{MPBY}^*_{t,t'} \), is the minimum across individuals \( i \) and histories \( h \) of the marginal propensity to spend in period-state \( (t, h) \) out of “certain” income across all histories \( h' \) in period \( t' \). This is analogous to the lower bound \( \text{MPBY}^*_{s,s'} \) in Equation S5. The second term of the lower bound, \( C_{t,t'} \), is a bias term. It is positive (negative) when the marginal propensity to spend is larger (smaller) out of histories \( h' \) in which compensated demand, and therefore consumer surplus, is larger. Terms for upper bounds are analogously defined, with additional details below.

I apply this decomposition to the bounds on period-state \( (t, h) \) income effects in Equation
S5, and derive the following bounds on period $t$ income effects.

\[
\sum_{t'=1}^{T} (1 + C_{t,t'}) \text{MPBY}_{t,t'}^{*} \text{MPBY}_{t',t}^{*} \mathbb{E}_{h'} \left[ \text{EV}_{(t',h')}^{*}(p^k, p^1; y, i) \mid i \right]
\]

\[
\leq p^k \left( B_{(t,h)}(p^k, y + \text{EV}^{*}(p^k, p^1; y, i); i) - B_{(t,h)}(p^k, y; i) \right)
\]

\[
\leq \sum_{t'=1}^{T} (1 + C_{t,t'}) \text{MPBY}_{t,t'}^{*} \text{MPBY}_{t',t}^{*} \mathbb{E}_{h'} \left[ \text{EV}_{(t',h')}^{*}(p^k, p^1; y, i) \mid i \right] \quad \text{(S8)}
\]

Bounds on the difference between compensated demand and demand in period-state $(t, h)$ are equal the sum, across periods $t'$, of bounds on the marginal propensity to spend out of certain income, $\text{MPBY}_{t,t'}^{*}$ and $\text{MPBY}_{t',t}^{*}$, times average consumer surplus in period $t'$, $\mathbb{E}_{h'} \left[ \text{EV}_{(t',h')}^{*}(p^k, p^1; y, i) \mid i \right]$, times bias terms.

Taking expectations of Equation S8 across households and differentiating yields

\[
\sum_{t'=1}^{T} (1 + C_{t,t'}) \text{MPBY}_{t,t'}^{*} \frac{\mathbb{E}_{i} \left[ \mathbb{E}_{h'} \left[ \text{EV}_{(t',h')}^{*}(p^k, y; i) \mid i \right] \right]}{\mathbb{E}_{i} \left[ \mathbb{E}_{h} \left[ B_{(t,h)}(p^1, y; i) \mid i \right] \right]} \leq \frac{p^1}{\mathbb{E}_{i} \left[ \mathbb{E}_{h} \left[ B_{(t,h)}(p^1, y; i) \mid i \right] \right]} \left( \frac{d \mathbb{E}_{i} \left[ \mathbb{E}_{h} \left[ B_{(t,h)}(p^1, y + \text{EV}^{*}(p^k, p^1; y, i); i) \mid i \right] \right]}{dp^k} \bigg|_{k=1} - \frac{d \mathbb{E}_{i} \left[ \mathbb{E}_{h} \left[ B_{(t,h)}(p^1, y; i) \mid i \right] \right]}{dp^1} \right)
\]

\[
\leq \sum_{t'=1}^{T} (1 + C_{t,t'}) \text{MPBY}_{t,t'}^{*} \frac{\mathbb{E}_{i} \left[ \mathbb{E}_{h'} \left[ \text{EV}_{(t',h')}^{*}(p^k, y; i) \mid i \right] \right]}{\mathbb{E}_{i} \left[ \mathbb{E}_{h} \left[ B_{(t,h)}(p^1, y; i) \mid i \right] \right]} \quad \text{(S9)}
\]

Equation S9 bounds the difference between the average (with respect to unobservable heterogeneity across households and states of the world) compensated and uncompensated price elasticities of demand. Identical to the bounds without unobserved heterogeneity in Equation 12, or with unobservable heterogeneity across households in Equation S6, these bounds depend on the growth rate of average investment expenditures, $\frac{\mathbb{E}_{i} \left[ \mathbb{E}_{h'} \left[ B_{(t',h')}^{*}(p^1, y; i) \mid i \right] \right]}{\mathbb{E}_{i} \left[ \mathbb{E}_{h} \left[ B_{(t,h)}(p^1, y; i) \mid i \right] \right]}$, and bounds on the marginal propensity to spend out of certain income, $\text{MPBY}_{t,t'}^{*}$ and $\text{MPBY}_{t',t}^{*}$. However, in contrast to the case when unobservable heterogeneity is only across households, there are additional adjustment factors $C_{t,t'}$ and $\overline{C}_{t,t'}$; these adjustment factors are 0 when marginal propensities to spend do not systematically differ out of income in histories $h'$ with respect to compensated demand. I implicitly assume these terms are 0, while using conservative bounds on marginal propensities to spend out of certain income, in the empirical analysis in Section 4.
ONLINE APPENDIX

Derivation of Equation S8 To derive the bounds in Equation S8, I first assume that the bounds in Equation S5 hold. Substituting \((t, h)\) for \(s\) and \((t', h')\) for \(s'\), and replacing inner products with sums, this implies

\[
\sum_{t'=1}^{T} \sum_{h'=1}^{H} MPBY_{(t,h),(t',h')} EV^*_{(t',h')}(p^k, p^1; y, i) \\
\leq p^k \left( B_{(t,h)}(p^k, y + EV^*(p^k, p^1; y, i); i) - B_{(t,h)}(p^k, y; i) \right) \\
\leq \sum_{t'=1}^{T} \sum_{h'=1}^{H} MPBY_{(t,h),(t',h')} EV^*_{(t',h')}(p^k, p^1; y, i) \quad (S10)
\]

Bounds on the marginal propensity to spend out of income in \((t', h')\) imply bounds on the marginal propensity to spend out of certain income in period \(t'\), which I denote \(MPBY^*_{(t,h),t'}\) and \(MPBY^*_{(t,h),t'}\). Bounds on the marginal propensity to spend out of certain income are the sum over the bounds on the effects of one unit of income across all possible histories \(h'\) in a given time period \(t'\).

\[
MPBY^*_{(t,h),t'} \equiv \sum_{h'=1}^{H} MPBY_{(t,h),(t',h')}
\]

\[
MPBY^*_{(t,h),t'} \equiv \sum_{h'=1}^{H} MPBY_{(t,h),(t',h')}
\]

Adding and subtracting average period \(t'\) equivalent variation \(\mathbb{E}_{h'} \left[ EV^*_{(t',h')}(p^k, p^1; y, i) \mid i \right] \) for each of the bounds in Equation S10 inside the sum across histories \(h'\), and substituting bounds on the marginal propensity to spend out of certain income, yields

\[
\sum_{t'=1}^{T} \left( \sum_{h'=1}^{H} MPBY_{(t,h),(t',h')} \left( EV^*_{(t',h')}(p^k, p^1; y, i) - \mathbb{E}_{h'} \left[ EV^*_{(t',h')}(p^k, p^1; y, i) \mid i \right] \right) \right) + \\
\leq p^k \left( B_{(t,h)}(p^k, y + EV^*(p^k, p^1; y, i); i) - B_{(t,h)}(p^k, y; i) \right) \\
\leq \sum_{t'=1}^{T} \left( \sum_{h'=1}^{H} MPBY_{(t,h),(t',h')} \left( EV^*_{(t',h')}(p^k, p^1; y, i) - \mathbb{E}_{h'} \left[ EV^*_{(t',h')}(p^k, p^1; y, i) \mid i \right] \right) \right) + \\
\leq \sum_{t'=1}^{T} \left( \sum_{h'=1}^{H} MPBY_{(t,h),(t',h')} \left( EV^*_{(t',h')}(p^k, p^1; y, i) - \mathbb{E}_{h'} \left[ EV^*_{(t',h')}(p^k, p^1; y, i) \mid i \right] \right) \right) + \\
MPBY^*_{(t,h),t'} \mathbb{E}_{h'} \left[ EV^*_{(t',h')}(p^k, p^1; y, i) \mid i \right] \quad (S11)
\]

7
I next define

\[
C^{**}_{(t,h),t',i} = \frac{\sum_{h'=1}^H \text{MPBY}^{*}_{(t,h),(t',h')} \left( \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) - \mathbb{E}_{h'} \left[ \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) \right] i \right)}{\sum_{h'=1}^H \text{MPBY}^{*}_{(t,h),(t',h')}} \mathbb{E}_{h'} \left[ \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) \right] i}
\]

\[
\overline{C}^{**}_{(t,h),t',i} = \frac{\sum_{h'=1}^H \text{MPBY}^{*}_{(t,h),(t',h')} \left( \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) - \mathbb{E}_{h'} \left[ \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) \right] i \right)}{\sum_{h'=1}^H \text{MPBY}^{*}_{(t,h),(t',h')}} \mathbb{E}_{h'} \left[ \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) \right] i}
\]

Let \( P_{h',t',i} \) denote the probability of history \( h' \) in period \( t' \) for individual \( i \). \( C^{**}_{(t,h),t',i} \) is a measure of the correlation for individual \( i \) between the bound on the inverse probability scaled marginal propensity to spend out of history \( h' \), \( \frac{1}{P_{h',t',i}} \text{MPBY}^{*}_{(t,h),(t',h')} \), and history \( h' \) equivalent variation \( \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) \). It is distinct from a correlation in that it is rescaled by the coefficient of variation across histories \( h' \) of equivalent variation, and the coefficient of variation across histories \( h' \) of the inverse probability scaled marginal propensity to spend. \( \overline{C}^{**}_{(t,h),t',i} \) is defined, and interpreted, analogously.

That marginal propensities to spend are scaled by the inverse probability of a history \( h' \) ensures that this definition is robust to “splitting” histories in a manner that does not affect observed behavior – for instance, splitting one history into two.

Note that when the correlation between bounds on inverse probability scaled marginal propensities to spend out of history \( h' \) and equivalent variation in history \( h' \) are 0, then \( C^{**}_{(t,h),t',i} = 0 \) and \( \overline{C}^{**}_{(t,h),t',i} = 0 \).

Substituting the definitions of \( C^{**}_{(t,h),t',i} \) and \( \overline{C}^{**}_{(t,h),t',i} \) into Equation S11 yields

\[
\sum_{t'=1}^T \left( 1 + C^{**}_{(t,h),t',i} \right) \text{MPBY}^{**}_{(t,h),t'} \mathbb{E}_{h'} \left[ \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) \right] i \right]
\]

\[
\leq p^k \left( B_{(t,h)} (p^k, y, EV^* (p^k, p^1; y, i); i) - B_{(t,h)} (p^k, y; i) \right)
\]

\[
\leq \sum_{t'=1}^T \left( 1 + \text{MPBY}^{**}_{(t,h),t'} \right) \text{MPBY}^{**}_{(t,h),t'} \mathbb{E}_{h'} \left[ \text{EV}^{*}_{(t',h')} (p^k, p^1; y, i) \right] i \right] \quad (S12)
\]

Define

\[
\text{MPBY}^*_{t,t'} \equiv \min_h \text{MPBY}^*_{(t,h),t'} \quad C^*_{t,t'} \equiv \frac{\min_{h,i} \left( 1 + C^{**}_{(t,h),t',i} \right) \text{MPBY}^{**}_{(t,h),t'}}{\min_h \text{MPBY}^{**}_{(t,h),t'}} - 1
\]

\[
\text{MPBY}^*_{t,t'} \equiv \max_h \text{MPBY}^*_{(t,h),t'} \quad \overline{C}^*_{t,t'} \equiv \frac{\max_{h,i} \left( 1 + \overline{C}^{**}_{(t,h),t',i} \right) \text{MPBY}^{**}_{(t,h),t'}}{\max_h \text{MPBY}^{**}_{(t,h),t'}} - 1
\]
The terms $\overline{\text{MPBY}}^{*}_{t,t'}$ and $\overline{\text{MPBY}}^{*}_{t,t'}$ are bounds, across histories $h$, on the marginal propensity to spend in period $t$ out of certain income in period $t'$. The terms $C_{t,t'}$ and $\overline{C}_{t,t'}$ are adjustments to these bounds that reflect the possibility that inverse probability scaled marginal propensities to spend out of particular histories may be higher out of histories with systematically higher or lower compensated demand.

Note that by construction, $(1 + C_{t,t'})\overline{\text{MPBY}}^{*}_{t,t'} \leq (1 + \overline{C}_{(t,h),t'}^{(t,h)},t') \overline{\text{MPBY}}^{*}_{(t,h),t'}$ and $(1 + \overline{C}_{t,t'})\overline{\text{MPBY}}^{*}_{t,t'} \geq (1 + \overline{C}_{(t,h),t'}^{(t,h),t'}) \overline{\text{MPBY}}^{*}_{(t,h),t'}$ for all histories $h$. As a consequence, for price decreases from $p^k$ to $p^1$ and therefore for positive equivalent variation,

$$\sum_{t'=1}^{T} (1 + C_{t,t'}) \overline{\text{MPBY}}^{*}_{t,t'} \mathbb{E}_{h'} \left[ \text{EV}^{*}_{(t',h')}(p^k, p^1; y, i) \bigg| i \right] \leq p^k \left( B_{(t,h)}(p^k, y + \text{EV}^{*}(p^k, p^1; y, i); i) - B_{(t,h)}(p^k, y; i) \right) \leq \sum_{t'=1}^{T} (1 + \overline{C}_{t,t'}) \overline{\text{MPBY}}^{*}_{t,t'} \mathbb{E}_{h'} \left[ \text{EV}^{*}_{(t',h')}(p^k, p^1; y, i) \bigg| i \right]$$

which is Equation S8.

**Online Appendix D  Ranking welfare impacts**

In the static setting, average consumer surplus is often used to rank policies that change income and prices – a policy with consumer surplus greater than costs passes a cost-benefit analysis. More generally, one policy is preferred to another if it achieves the same consumer surplus for a group of households at lower costs, or if it more efficiently transfers consumer surplus across groups of households (Finkelstein & Hendren, 2020). However, the use of average consumer surplus to rank policies has well known limitation: for a higher average consumer surplus policy to necessarily enable a Pareto improvement, it must be the case that any lump-sum transfer scheme is possible across households.$^A$ Unobservably unobservable heterogeneity across households in general leaves Pareto improvements implausible for many policies that increase average surplus (Sallee, 2022).$^B$

In a stochastic dynamic model with complete financial markets, ranking policies that change income and prices is a natural extension of the static setting using the net present value (NPV) of average consumer surplus. Consumer surplus can be aggregated within

$^A$Note that a household here implicitly corresponds to $(i, h)$, that is both a household and a potential history of shocks.

$^B$This is distinct from the concern that transfers targeted on observables may be costly, with costs potentially including those from behavioral responses to the transfers. In this case, appropriately weighted average consumer surplus can be used (Hendren, 2020).
Online Appendix

Online Appendix E  Consumption impacts of and consumer surplus from microfinance

The impacts of microfinance on household consumption may be biased upwards or downwards for consumer surplus (Banerjee et al., 2015b); in this section, I compare the impacts of microfinance on household consumption to consumer surplus, and I decompose bias into two channels. The first is the “consumption smoothing” channel – consumption impacts are biased downwards for consumer surplus, as they do not overweight impacts on consumption in high marginal utility of consumption period-states. The second is the “risky investment” channel – consumption impacts are biased upwards for consumer surplus, as they do not discount uncertain investment returns. I express each of these channels as a function of estimable sufficient statistics, for which I discuss plausible values. I do so in a stylized model; by necessity, this abstracts from some important determinants of the consumption impacts of microfinance, including ex-ante consumption responses (Kaboski & Townsend, 2011).

I evaluate bias in impacts of microfinance on household consumption as a measure of

\[ A4 \]

Farhi et al. (2022) note that a social planner who can implement arbitrary targeted transfers within period-state, and also faces complete financial markets, could themselves complete financial markets and increase social welfare. That this does not happen in practice relates to the implausibility of general lump-sum transfer schemes targeted on unobservables (Sallee, 2022).
consumer surplus from small price changes in a simplified version of the model in Section 2. Households have access to two investment technologies – microfinance loans, and a risky investment technology. Households choose consumption, microfinance loans, and risky investments in period 1, and in the next period consume the output of their risky investments net of loan repayments. Specifically, households solve

\[
V(p, y) = \max_{c, b_x, b_{x, h}} u(c_1) + \beta \sum_{h=1}^{H} P_h u(c_{2,h})
\]

subject to
\[
c_1 + pb_x + b_x \leq y_1
\]
\[
c_{2,h} \leq y_{2,h} + (1 + r_{x,h})b_x - b_{\ell} \quad \forall h \in \{1, \ldots, H\}
\]

The model in Equation S13 is a special case of Equation 1. Utility is discounted expected utility with utility index \(u\), where \(P_h\) is the probability of state \(h\). In period 1, the household faces the constraint that consumption \(c_1\) plus risky investments \(b_x\) is equal to income \(y_1\) plus loan amount \(-pb_{\ell}\). In period 2, in state \(h \in \{1, \ldots, H\}\), the household consumes its period-state income \(y_{2,h}\) and its investment plus risky returns \((1 + r_{x,h})b_x\), net of loan repayments \(b_{\ell}\).

I define the impacts of a small price change on consumption to be expected impacts on the sum of period 1 and next period consumption; this is consistent with empirical analysis of the impacts of microfinance which pools across both current and past borrowers. I use \(\mathbb{E}_h\) to denote expectation across states \(h \in \{1, \ldots, H\}\) with respect to \(P\); with this notation, the impacts of a small price change on consumption are \(\frac{dC_1(p, y) + \mathbb{E}_h[C_{2,h}(p, y)]}{dp}\). Furthermore, I define \(r_{\ell} = -\frac{p+1}{p}\) to be the interest rate on credit; note that one plus the interest rate on credit \(1 + r_{\ell}\) times loan amount \(-pb_{\ell}\) is equal to repayment obligations \(b_{\ell}\). Using the period-state budget constraints in Equation S13, and applying Roy’s identity (Equation 4), I derive

\[
\frac{d(C_1(p, y) + \mathbb{E}_h[C_{2,h}(p, y)])}{dp} / (dV(p, y)/dp) / (dV(p, y)/dy_1) = 1 + r_{\ell} + \epsilon \text{MPXL} [\mathbb{E}_h[r_{x,h}] - r_{\ell}] - \epsilon(1 - \text{MPXL}) r_{\ell}
\]

The impacts of a small price change on consumption decompose into three terms.

The first is the direct effect \(1 + r_{\ell}\), the effect of the price change on consumption holding fixed loan amount.

The second is the risky investment effect, which is the product of three terms – the price elasticity of expenditures \(\epsilon \equiv \frac{B_{1}(p, y) + dB_{1}(p, y)/dp}{B_{1}(p, y)}\), the marginal propensity to invest out of marginal borrowing \(\text{MPXL} \equiv \frac{dB_{x}(p, y)/dp}{B_{1}(p, y) + pdB_{1}(p, y)/dp}\), and the difference between average returns
on risky investments and the interest rate on credit, \( E_h[r_{x,h}] - r_t \). While there is no first order welfare effect of marginal risky investments relative to consumption, the consumption impacts of marginal risky investment are equal to the increased risky investments caused by the price change, times the difference between expected returns on investment and interest on credit. Consumption impacts weight the returns to risky investments equally across periods and states, while households downweight these returns both due to impatience \( \beta \) and concavity in utility \( u \); as a result, risky investment effects cause consumption impacts to be biased upwards for consumer surplus.

The third is the consumption smoothing effect, which is the product of three terms – the price elasticity of expenditures \( \epsilon \), the marginal propensity to consume out of marginal borrowing \( 1 - MPXL \), and the negative of the interest rate on credit \( -r_t \). While there is no first order welfare effect of marginal consumption relative to risky investments, the consumption impacts of marginal consumption are equal to the increased consumption caused by the price change, times the interest on credit financing that consumption. Consumption impacts weight present and future consumption equally, while households downweight future consumption; as a result, consumption smoothing effects cause consumption impacts to be biased downwards for consumer surplus.

Note that when borrowing does not respond to price changes, that is \( \epsilon = 1 \), and the full income effect of the price change is consumed, that is \( MPXL = 0 \), then consumption impacts are equal to consumer surplus. In this case, the price change manifests as a dollar-for-dollar increase in period 1 consumption, and the corresponding transfer causes identical first order impacts on welfare.

In general, consumption impacts need not be equal to impacts on consumer surplus. I calibrate Equation S14 based on the lending product in Section 4.3; I substitute \( r_t = 0.207 \), corresponding to total repayment obligations associated with a 4.5% monthly add-on interest rate, and \( E_h[r_{x,h}] = 0.44 \), corresponding to the lower end of estimates of two month (half the repayment period) returns on investment from McKenzie & Woodruff (2008) in Mexico. When half of marginal loans are invested, that is \( MPXL = 0.5 \), Year 1 and Year 3 price elasticities of expenditures from Table 2 correspond to ratios of consumption impacts to consumer surplus of 1.3 and 1.4. In this case, consumption impacts are relatively close to consumer surplus, as risky investment effects of 0.75 and 1.64 are offset by consumption smoothing effects of -0.66 and -1.46. However, when three quarters of marginal loans are invested, that is \( MPXL = 0.75 \), Year 1 and Year 3 price elasticities of expenditures from Table 2 correspond to ratios of consumption impacts to consumer surplus of 2.1 and 3.2; risky investment effects dominate as most of marginal borrowing is invested.