Factor Market Failures and the Adoption of Irrigation in Rwanda*

ONLINE APPENDIX

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Appendix A Appendix Tables

Table A1: Access to irrigation decreases dry season NDVI, suggesting decreases in banana cultivation

	SP, Landsat, Discontinuity sample					
	Year≤ 2008		Year≥ 2015			
	100 * NDVI					
	(1)	(2)	(3)	(4)	(5)	(6)
SP CA	0.300		-0.214			
	(0.251)		(0.250)			
	[0.235]		[0.395]			
Rainy seasons * SP CA		0.385		0.045		
		(0.258)		(0.250)		
		[0.138]		[0.859]		
Dry season * SP CA		0.098		-0.578		
		(0.279)		(0.289)		
		[0.727]		[0.047]		
SP Banana					0.638	
					(0.189)	
					[0.001]	
Rainy seasons * SP Banana						0.315
						(0.186)
						[0.093]
Dry season * SP Banana						1.097
						(0.210)
						[0.000]
Joint F-stat [p]	1.4	1.9	0.7	4.9	11.4	18.4
	[0.235]	[0.151]	[0.394]	[0.008]	[0.001]	[0.000]
Pre-construction (Year ≤2008)	X	X				
Post-construction (Year ≥2015)			X	X	X	X
Site-by-image FE	X	X	X	X	X	X
SP distance to boundary	X	X	X	X		
SP log GPS area	X	X	X	X		
Dry season * SP distance to boundary		X		X		
Dry season * SP log GPS area		X		X		
# of observations	54,533	54,533	29,458	29,458	27,507	27,507
# of clusters	249	249	174	174	173	173

Table A2: Sample plot access to irrigation increases largest other plot NDVI, suggesting increases in banana cultivation

	LOP, Landsat, Discontinuity sample			
	Year≤ 2008		Year≥ 2015	
	100 * NDVI			
	(1)	(2)	(3)	(4)
SP CA	0.204		0.354	
	(0.275)		(0.223)	
	[0.459]		[0.114]	
Rainy seasons * SP CA		0.296		0.486
		(0.293)		(0.241)
		[0.314]		[0.046]
Dry season * SP CA		-0.016		0.162
		(0.293)		(0.247)
		[0.957]		[0.512]
Joint F-stat [p]	0.6	1.2	2.5	2.4
	[0.458]	[0.315]	[0.114]	[0.096]
Pre-construction (Year ≤2008)	X	X		
Post-construction (Year ≥2015)			X	X
Site-by-image FE	X	X	X	X
SP distance to boundary	X	X	X	X
SP log GPS area	X	X	X	X
LOP log GPS area	X	X	X	X
LOP CA	X	X	X	X
Dry season * SP distance to boundary		X		X
Dry season * SP log GPS area		X		X
Dry season * LOP log GPS area		X		X
Dry season * LOP CA		X		X
# of observations	44,635	44,635	23,828	23,828
# of clusters	282	282	165	165

Table A3: Terracing, baseline rentals to commercial farmer, and elevation in command area

	SP, Baseline, Discontinuity sample		
	Terraced	Rented out, comm. farmer	Elevation
	(1)	(2)	(3)
RDD (Site FE, Specification 1)			
SP CA	0.401	0.180	-21.9
	(0.055)	(0.033)	(4.3)
	[0.000]	[0.000]	[0.000]
SFE (Spatial FE, Specification 2)			
SP CA	0.445	0.176	-8.9
	(0.054)	(0.044)	(1.2)
	[0.000]	[0.000]	[0.000]
# of observations	931	931	931
# of clusters	174	174	174
Control mean	0.478	0.018	1741.9

Table A4: Access to irrigation in the command area is limited at baseline

(a) Dry season

	SP, Baseline, Dry season, Discontinuity sample										
	Culti-vated	Irri-gated	Horti-culture	Banana	HH labor/ ha	Input exp./ha	Hired labor exp./ha	Yield	Sales /ha	Profits/ha Shadow wage = 0	= 800
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
RDD (Site-by-season FE, Specification 1)											
SP CA	-0.124	0.032	0.023	-0.105	-26.3	1.9	1.2	-26.7	-23.0	-28.7	-14.7
	(0.046)	(0.016)	(0.019)	(0.036)	(25.4)	(2.2)	(1.5)	(23.6)	(21.8)	(22.3)	(13.7)
	[0.007]	[0.045]	[0.236]	[0.004]	[0.301]	[0.397]	[0.449]	[0.258]	[0.292]	[0.198]	[0.283]
SFE (Spatial FE, Specification 2)											
SP CA	-0.117	0.031	0.016	-0.080	-40.0	1.5	0.3	-29.1	-35.4	-31.0	-6.8
	(0.052)	(0.015)	(0.018)	(0.042)	(30.7)	(2.1)	(1.6)	(30.8)	(29.0)	(29.7)	(19.3)
	[0.024]	[0.043]	[0.371]	[0.055]	[0.192]	[0.459]	[0.869]	[0.345]	[0.221]	[0.296]	[0.726]
# of observations	856	856	856	856	852	856	856	831	856	831	827
# of clusters	173	173	173	173	173	173	173	172	173	172	172
Control mean	0.207	0.006	0.009	0.146	42.7	1.9	0.5	45.1	25.6	43.8	11.5

(b) Rainy seasons

	SP, Baseline, Rainy seasons, Discontinuity sample										
	Culti-vated	Irri-gated	Horti-culture	Banana	HH labor/ ha	Input exp./ha	Hired labor exp./ha	Yield	Sales /ha	Profits/ha Shadow wage = 0	= 800
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
RDD (Site-by-season FE, Specification 1)											
SP CA	-0.062	0.044	0.053	-0.106	-2.4	2.3	2.4	9.0	10.0	4.4	4.0
	(0.040)	(0.011)	(0.022)	(0.037)	(23.8)	(3.4)	(4.3)	(23.1)	(14.1)	(23.6)	(24.3)
	[0.119]	[0.000]	[0.013]	[0.004]	[0.919]	[0.490]	[0.569]	[0.697]	[0.478]	[0.852]	[0.868]
SFE (Spatial FE, Specification 2)											
SP CA	-0.042	0.042	0.062	-0.096	-5.7	4.0	2.6	-2.5	24.5	-8.7	-7.2
	(0.043)	(0.015)	(0.029)	(0.039)	(35.5)	(3.9)	(5.9)	(29.4)	(18.1)	(29.3)	(35.4)
	[0.330]	[0.005]	[0.034]	[0.013]	[0.872]	[0.297]	[0.662]	[0.933]	[0.175]	[0.766]	[0.839]
# of observations	1,550	1,550	1,550	1,550	1,541	1,550	1,550	1,507	1,550	1,507	1,499
# of clusters	169	169	169	169	169	169	169	169	169	169	169
Control mean	0.752	0.011	0.037	0.164	226.9	12.6	12.4	173.4	45.0	148.4	-28.4

Table A5: Sample plot attrition

	Discontinuity sample, Dry season			Discontinuity sample, Rainy seasons		
	Dep. var. mean (Dep. var. SD) # of obs.	SP CA Coef. (SE) [p-value]		Dep. var. mean (Dep. var. SD) # of obs.	SP CA Coef. (SE) [p-value]	
	(1)	(2)	(3)	(4)	(5)	(6)
Dep. var. (SP)						
Tracked	0.033	0.022	0.037	0.048	0.020	0.037
	(0.178)	(0.014)	(0.019)	(0.215)	(0.016)	(0.023)
	2,793	[0.109]	[0.056]	4,655	[0.217]	[0.102]
Missing	0.058	0.120	0.098	0.061	0.115	0.092
	(0.233)	(0.025)	(0.028)	(0.240)	(0.025)	(0.028)
	2,793	[0.000]	[0.000]	4,655	[0.000]	[0.001]
Reason data is missing						
HH attrition	0.036	0.028	0.032	0.036	0.028	0.033
	(0.186)	(0.019)	(0.022)	(0.187)	(0.019)	(0.022)
	2,793	[0.135]	[0.148]	4,655	[0.136]	[0.143]
Rented out comm. farmer	0.012	0.091	0.067	0.011	0.087	0.063
	(0.109)	(0.019)	(0.015)	(0.106)	(0.019)	(0.015)
	2,793	[0.000]	[0.000]	4,655	[0.000]	[0.000]
Transaction (not tracked)	0.010	0.002	-0.001	0.014	0.000	-0.003
	(0.099)	(0.005)	(0.007)	(0.116)	(0.006)	(0.008)
	2,793	[0.741]	[0.852]	4,655	[0.984]	[0.681]
Controls						
Site-by-season FE		X			X	
SP distance to boundary		X	X		X	X
SP log GPS area		X	X		X	X
Spatial FE			X			X

Table A6: Effects of access to irrigation on sample plots and effects of access to irrigation on largest other plots are similar

(a) Effects of access to irrigation on sample plots on sample plot and largest other plot adoption of irrigation

	Dry season, SP discontinuity sample		
	Sample plot	Largest other plot	
	Irrigated		
	(1)	(2)	(3)
RDD (Site-by-season FE, Specifications 1 & 3)			
SP CA	0.163	-0.049	-0.002
	(0.024)	(0.026)	(0.020)
	[0.000]	[0.055]	[0.921]
SP CA * LOP CA			-0.112
			(0.035)
			[0.002]
Joint F-stat [p]	45.8	3.7	5.0
	[0.000]	[0.057]	[0.008]
SFE (Spatial FE, Specifications 2 & 4)			
SP CA	0.177	-0.041	0.018
	(0.030)	(0.032)	(0.025)
	[0.000]	[0.206]	[0.464]
SP CA * LOP CA			-0.131
			(0.045)
			[0.003]
Joint F-stat [p]	35.0	1.6	4.4
	[0.000]	[0.206]	[0.012]
# of observations	2,439	2,107	2,107
# of clusters	173	165	165

(b) Effects of access to irrigation on largest other plots on sample plot and largest other plot adoption of irrigation

	Dry season, LOP discontinuity sample		
	Largest other plot	Sample plot	
	Irrigated		
	(1)	(2)	(3)
RDD (Site-by-season FE, Specifications 1 & 3)			
LOP CA	0.148	-0.044	0.004
	(0.026)	(0.036)	(0.036)
	[0.000]	[0.211]	[0.908]
LOP CA * SP CA			-0.079
			(0.044)
			[0.077]
Joint F-stat [p]	31.4	1.6	1.8
	[0.000]	[0.212]	[0.171]
SFE (Spatial FE, Specifications 2 & 4)			
LOP CA	0.183	-0.071	-0.006
	(0.029)	(0.041)	(0.043)
	[0.000]	[0.087]	[0.883]
LOP CA * SP CA			-0.109
			(0.049)
			[0.026]
Joint F-stat [p]	39.5	2.9	3.1
	[0.000]	[0.087]	[0.044]
# of observations	1,502	1,460	1,460
# of clusters	158	154	154

Table A7: No effects of sample plot shock on largest other plots at baseline

	LOP, Baseline, Dry season, Discontinuity sample						
	Cultivated	Irrigated	Horticulture	Banana	HH labor/ ha	Input exp./ha	Hired labor exp./ha
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
RDD (Site-by-season FE, Specification 3)							
SP CA	0.025	0.020	0.016	0.027	-12.5	3.2	-7.5
	(0.048)	(0.018)	(0.017)	(0.038)	(21.3)	(1.5)	(4.2)
	[0.606]	[0.263]	[0.341]	[0.469]	[0.556]	[0.040]	[0.079]
SFE (Spatial FE, Specification 4)							
SP CA	0.012	0.003	0.013	0.043	-41.7	-0.1	-11.9
	(0.058)	(0.019)	(0.016)	(0.045)	(29.2)	(1.3)	(5.8)
	[0.830]	[0.891]	[0.407]	[0.344]	[0.153]	[0.953]	[0.042]
# of observations	725	725	725	725	721	725	725
# of clusters	165	165	165	165	165	165	165
Control mean	0.187	0.031	0.027	0.130	41.8	1.4	5.2

Table A8: No effects of sample plot shock on largest other plots at baseline

	LOP, Baseline, Dry season, Discontinuity sample						
	Cultivated	Irrigated	Horticulture	Banana	HH labor/ ha	Input exp./ha	Hired labor exp./ha
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
RDD (Site-by-season FE, Specification 3)							
SP CA	0.007	0.022	0.021	0.006	-4.9	3.9	-6.3
	(0.059)	(0.016)	(0.016)	(0.050)	(27.6)	(2.0)	(5.4)
	[0.908]	[0.175]	[0.201]	[0.905]	[0.858]	[0.055]	[0.248]
SP CA * LOP CA	0.043	-0.004	-0.012	0.051	-18.2	-1.7	-2.8
	(0.063)	(0.031)	(0.030)	(0.050)	(28.4)	(3.2)	(4.9)
	[0.494]	[0.901]	[0.690]	[0.310]	[0.523]	[0.588]	[0.564]
Joint F-stat [p]	0.4	0.9	0.8	1.2	0.6	2.3	2.6
	[0.653]	[0.391]	[0.441]	[0.306]	[0.572]	[0.107]	[0.078]
Sample plot effect	-0.124	0.032	0.023	-0.105	-26.3	1.9	1.2
Average effect	0.025	0.020	0.016	0.027	-12.5	3.2	-7.5
SFE (Spatial FE, Specification 4)							
SP CA	0.034	0.007	0.024	0.049	-27.5	1.5	-10.2
	(0.068)	(0.018)	(0.015)	(0.057)	(33.4)	(1.2)	(6.9)
	[0.614]	[0.678]	[0.109]	[0.388]	[0.411]	[0.206]	[0.142]
SP CA * LOP CA	-0.049	-0.011	-0.025	-0.014	-31.7	-3.5	-3.7
	(0.071)	(0.030)	(0.031)	(0.059)	(29.7)	(3.7)	(5.8)
	[0.490]	[0.723]	[0.420]	[0.816]	[0.287]	[0.349]	[0.522]
Joint F-stat [p]	0.2	0.1	1.3	0.5	1.8	0.8	2.7
	[0.779]	[0.888]	[0.274]	[0.637]	[0.167]	[0.440]	[0.064]
Sample plot effect	-0.117	0.031	0.016	-0.080	-40.0	1.5	0.3
Average effect	0.012	0.003	0.013	0.043	-41.7	-0.1	-11.9
# of observations	725	725	725	725	721	725	725
# of clusters	165	165	165	165	165	165	165
Control mean	0.187	0.031	0.027	0.130	41.8	1.4	5.2

Table A9: No average effects of sample plot shock on largest other plots during rainy season except increased cultivation of bananas

	LOP, Rainy seasons, Discontinuity sample						
	Culti-vated	Irri-gated	Horti-culture	Banana	HH labor/ha	Input exp./ha	Hired labor exp./ha
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
RDD (Site-by-season FE, Specification 3)							
SP CA	0.093	-0.005	-0.007	0.111	-0.3	-3.5	0.5
	(0.027)	(0.011)	(0.019)	(0.035)	(17.5)	(3.0)	(4.4)
	[0.001]	[0.639]	[0.723]	[0.002]	[0.988]	[0.245]	[0.902]
Sample plot effect	-0.085	0.038	0.025	-0.164	15.5	2.0	3.5
SFE (Spatial FE, Specification 4)							
SP CA	0.083	-0.004	-0.004	0.105	6.6	-2.3	4.6
	(0.028)	(0.015)	(0.022)	(0.039)	(19.5)	(3.7)	(5.0)
	[0.003]	[0.800]	[0.868]	[0.007]	[0.733]	[0.527]	[0.360]
Sample plot effect	-0.052	0.062	0.053	-0.170	10.4	2.2	3.1
# of observations	3,526	3,526	3,526	3,526	3,505	3,510	3,510
# of clusters	165	165	165	165	165	165	165
Control mean	0.858	0.027	0.070	0.228	209.3	16.3	18.6

Table A10: No heterogeneous effects of sample plot shock on largest other plots during rainy season with respect to location of largest other plots

	LOP, Rainy seasons, Discontinuity sample						
	Cultivated	Irrigated	Horticulture	Banana	HH labor/ ha	Input exp./ha	Hired labor exp./ha
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
RDD (Site-by-season FE, Specification 3)							
SP CA	0.096	-0.000	-0.000	0.110	-2.1	-0.4	-3.4
	(0.031)	(0.010)	(0.016)	(0.045)	(22.7)	(3.3)	(5.0)
	[0.002]	[0.986]	[0.988]	[0.015]	[0.927]	[0.903]	[0.502]
SP CA * LOP CA	-0.006	-0.012	-0.016	0.002	4.3	-7.4	9.3
	(0.033)	(0.015)	(0.026)	(0.047)	(27.7)	(3.8)	(5.6)
	[0.866]	[0.447]	[0.549]	[0.969]	[0.876]	[0.048]	[0.100]
Joint F-stat [p]	6.1	0.3	0.2	5.5	0.0	2.4	1.4
	[0.003]	[0.739]	[0.835]	[0.005]	[0.988]	[0.095]	[0.258]
Sample plot effect	-0.085	0.038	0.025	-0.164	15.5	2.0	3.5
Average effect	0.093	-0.005	-0.007	0.111	-0.3	-3.5	0.5
SFE (Spatial FE, Specification 4)							
SP CA	0.074	-0.001	0.007	0.111	-6.7	-0.3	-1.4
	(0.030)	(0.012)	(0.021)	(0.049)	(24.9)	(4.0)	(5.7)
	[0.013]	[0.918]	[0.756]	[0.022]	[0.789]	[0.937]	[0.806]
SP CA * LOP CA	0.020	-0.006	-0.023	-0.013	29.4	-4.4	13.2
	(0.035)	(0.016)	(0.028)	(0.053)	(30.2)	(4.6)	(7.6)
	[0.574]	[0.737]	[0.423]	[0.813]	[0.331]	[0.334]	[0.081]
Joint F-stat [p]	4.4	0.1	0.3	3.7	0.6	0.6	1.8
	[0.012]	[0.943]	[0.720]	[0.024]	[0.568]	[0.549]	[0.163]
Sample plot effect	-0.052	0.062	0.053	-0.170	10.4	2.2	3.1
Average effect	0.083	-0.004	-0.004	0.105	6.6	-2.3	4.6
# of observations	3,526	3,526	3,526	3,526	3,505	3,510	3,510
# of clusters	165	165	165	165	165	165	165
Control mean	0.858	0.027	0.070	0.228	209.3	16.3	18.6

Table A11: Household welfare

	HH, Discontinuity sample			
	Housing expenditures	Asset index	Food security index	Overall index
	(1)	(2)	(3)	(4)
RDD (Site-by-survey FE, Specification 1)				
SP CA	11.90	0.13	0.07	0.11
	(6.77)	(0.11)	(0.08)	(0.07)
	[0.079]	[0.246]	[0.424]	[0.090]
SFE (Spatial FE, Specification 2)				
SP CA	11.75	0.06	0.07	0.10
	(7.80)	(0.13)	(0.10)	(0.08)
	[0.132]	[0.666]	[0.503]	[0.216]
# of observations	2,666	2,672	2,668	2,659
# of clusters	173	173	173	173
Control mean	27.96	-0.10	-0.13	-0.08

Table A12: Household size and wealth shift agricultural production decisions in a manner consistent with them shifting the shadow wage and shadow price of inputs

	HH labor/ha				Input exp./ha				Hired labor exp./ha			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
# of HH members	7.4		5.0	4.7	0.5		-0.1	-0.0	0.6		-1.1	-0.9
	(1.2)		(1.8)	(1.5)	(0.2)		(0.3)	(0.2)	(0.3)		(0.4)	(0.4)
	[0.000]		[0.004]	[0.002]	[0.010]		[0.840]	[0.951]	[0.077]		[0.013]	[0.028]
Asset index		10.0	4.8	2.5		2.5	1.8	1.5		8.8	8.9	8.4
		(2.7)	(3.2)	(2.7)		(0.4)	(0.5)	(0.4)		(0.7)	(0.8)	(0.7)
		[0.000]	[0.129]	[0.367]		[0.000]	[0.000]	[0.000]		[0.000]	[0.000]	[0.000]
log area	-117.6	-117.4	-118.1	-120.1	-5.3	-5.5	-5.5	-6.1	-3.7	-4.7	-4.5	-5.1
	(4.0)	(4.1)	(4.1)	(3.7)	(0.5)	(0.5)	(0.5)	(0.4)	(0.5)	(0.5)	(0.6)	(0.5)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
# of HH members (15-64)			2.4	2.7			-0.3	-0.4			-0.6	-1.0
			(2.4)	(2.1)			(0.4)	(0.3)			(0.7)	(0.6)
			[0.329]	[0.198]			[0.417]	[0.191]			[0.346]	[0.107]
HHH female			-2.4	4.3			-3.1	-2.0			-0.0	0.7
			(6.4)	(5.8)			(0.9)	(0.8)			(1.4)	(1.3)
			[0.706]	[0.456]			[0.000]	[0.008]			[0.995]	[0.603]
# of plots			-1.4	-0.2			-0.1	0.1			0.2	0.3
			(0.9)	(0.8)			(0.2)	(0.1)			(0.2)	(0.2)
			[0.145]	[0.849]			[0.691]	[0.428]			[0.378]	[0.218]
Site-by-season FE	X	X	X	X	X	X	X	X	X	X	X	X
Site-by-season-by-crop FE				X				X				X
# of observations	28,750	28,717	28,578	28,576	28,823	28,790	28,651	28,649	28,823	28,790	28,651	28,649
# of clusters	1,637	1,635	1,628	1,628	1,637	1,635	1,628	1,628	1,637	1,635	1,628	1,628

Table A13: Interventions targeting operations and maintenance, land taxes, and access to inputs did not increase adoption of irrigation

	Farmer monitor			Land tax subsidies			Assigned minikit		
	Days w/o enough water	Days irrigated	Irrigated	Taxes owed	Taxes paid	Irrigated	Minikit takeup	Horticulture	Irrigated
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Farmer monitor	0.42	1.12	0.031						
	(0.87)	(1.68)	(0.023)						
	[0.632]	[0.506]	[0.184]						
Subsidy				-7.77	-0.22	0.024			
				(0.83)	(0.06)	(0.025)			
				[0.000]	[0.000]	[0.332]			
Assigned minikit							0.398	0.035	-0.005
							(0.038)	(0.041)	(0.040)
							[0.000]	[0.396]	[0.897]
Minikit saturation							-0.047	-0.078	-0.115
							(0.056)	(0.054)	(0.058)
							[0.394]	[0.149]	[0.049]
Sample (Plots)	SP in Command Area								
Sample (Seasons)	2016, 2017, & 2018 Dry			2017 Rainy 1 & 2		2017 & 2018 Dry	2017 Rainy 1 & Dry	2017 & 2018 Dry	
Minikit saturation	X	X	X	X	X	X			
Zone FE	X	X	X	X	X	X	X	X	X
# of subsidy lotteries entered				X	X	X			
# of lotteries entered							X	X	X
O&M treatment							X	X	X
# of observations	640	709	2,277	315	309	1,007	910	838	839
# of clusters	145	150	215	101	98	181	187	182	182
Control mean	4.45	32.41	0.294	9.62	0.47	0.319	0.061	0.331	0.369

Appendix B Appendix Figures

Figure A1: Karongi 12 hillside irrigation scheme

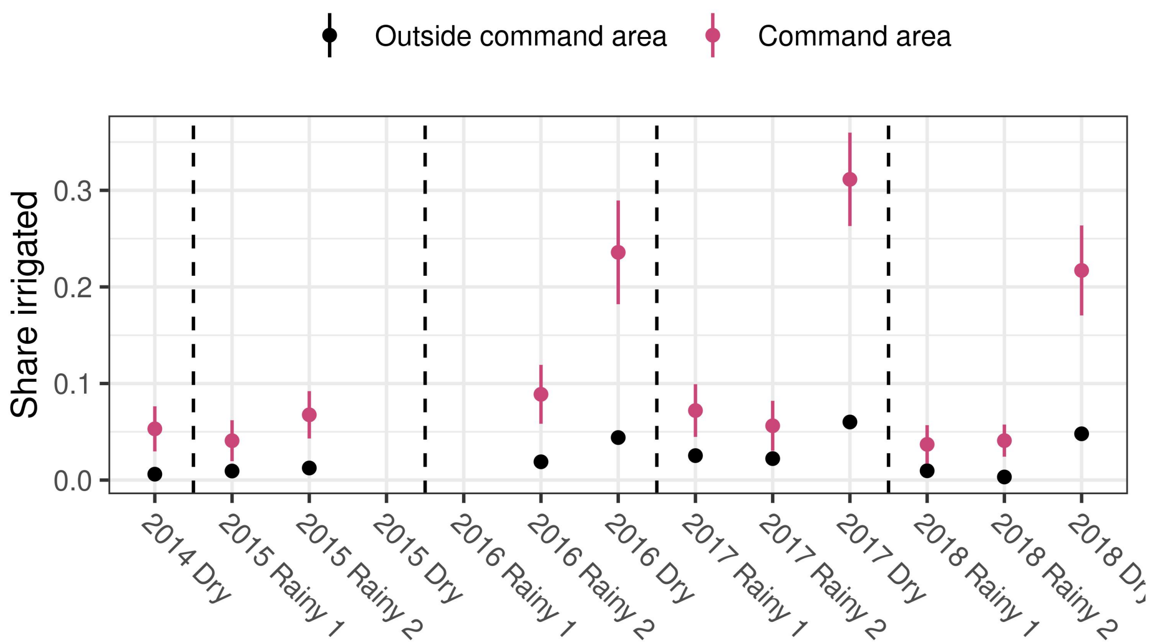


Figure A2: Adoption dynamics

Notes: Average adoption of irrigation by season on sample plots in the discontinuity sample, inside and outside the command area, is presented in this figure. Averages outside the command area are in black, while averages inside the command area and 95% confidence intervals for the difference are in pink. Robust standard errors are clustered at the nearest Water User Group level.

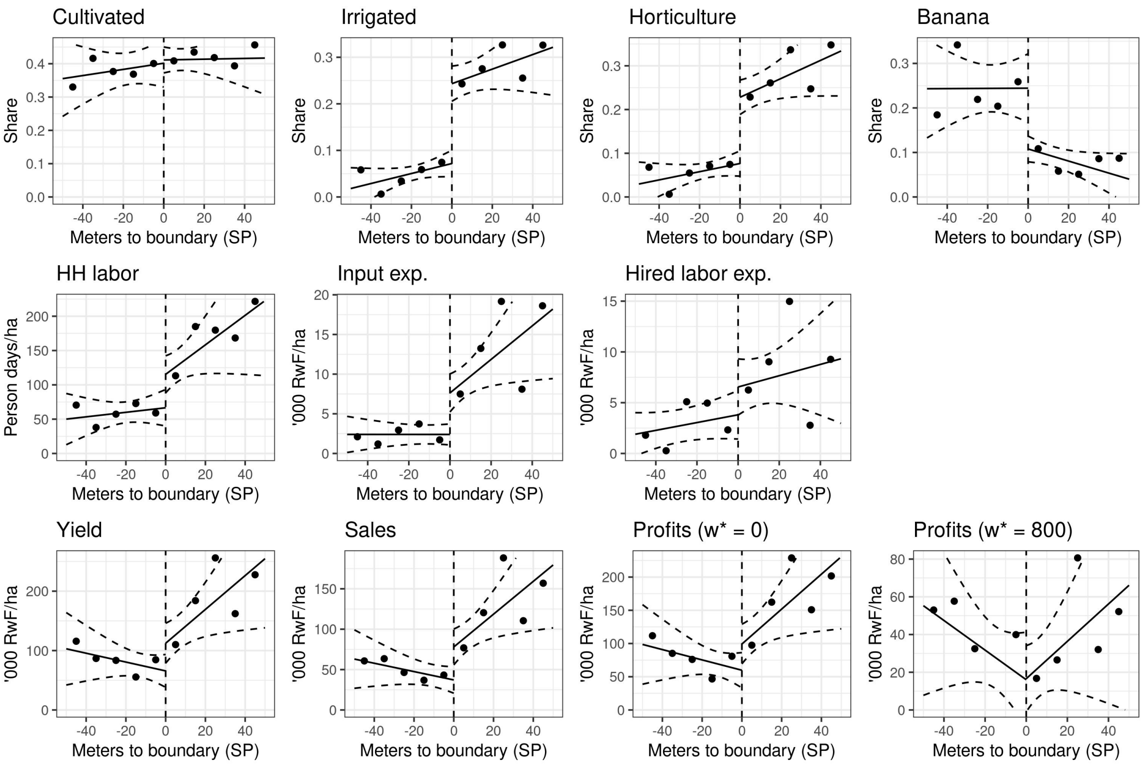


Figure A3: Regression discontinuity estimates of impacts of irrigation

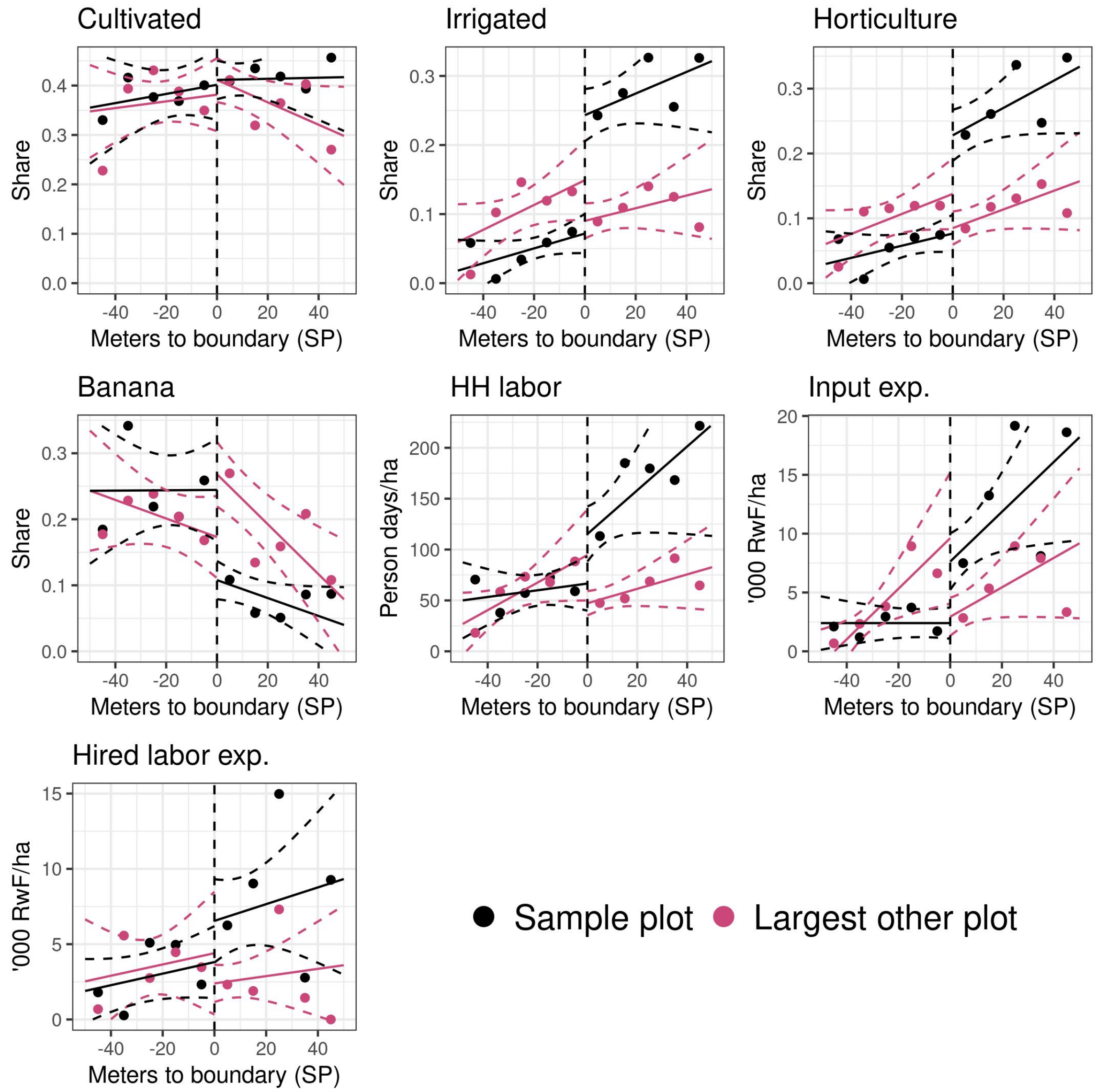


Figure A4: Regression discontinuity estimates of largest other plot responses to sample plot shock

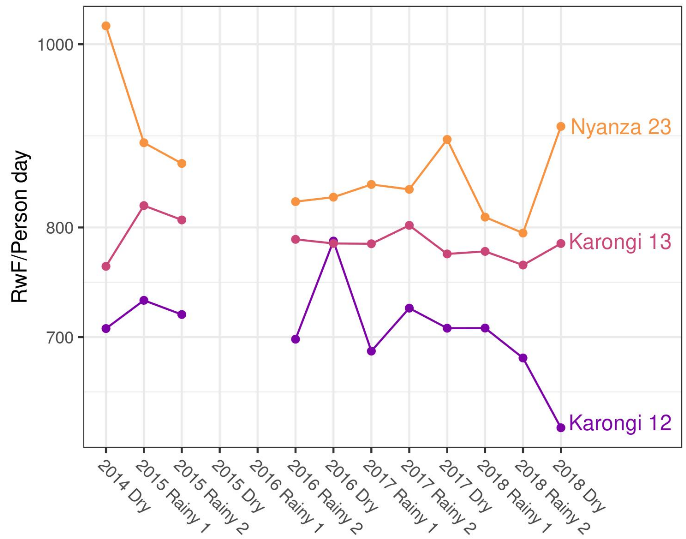
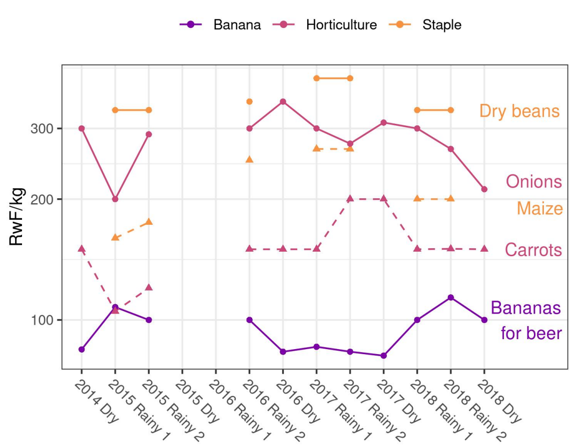
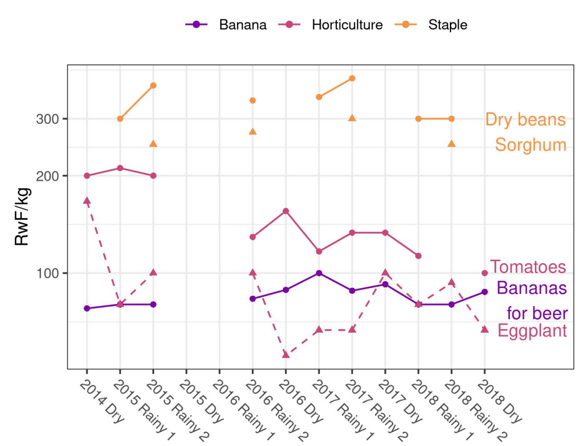


Figure A5: Wages

Notes: Average wages by season across the three hillside irrigation schemes are presented in this figure. Average wages are calculated across household-by-plot-by-season observations within site-by-season and are weighted by person days of hired labor.



(a) Karongi



(b) Nyanza

Figure A6: Prices

Notes: Median sale prices by season are presented in this figure. Prices are calculated separately for Karongi district (Karongi 12 and Karongi 13) and for Nyanza district (Nyanza 23). For each district, prices are calculated for the most commonly sold banana crop, the two most commonly sold staple crops, and the two most commonly sold horticultural crops.

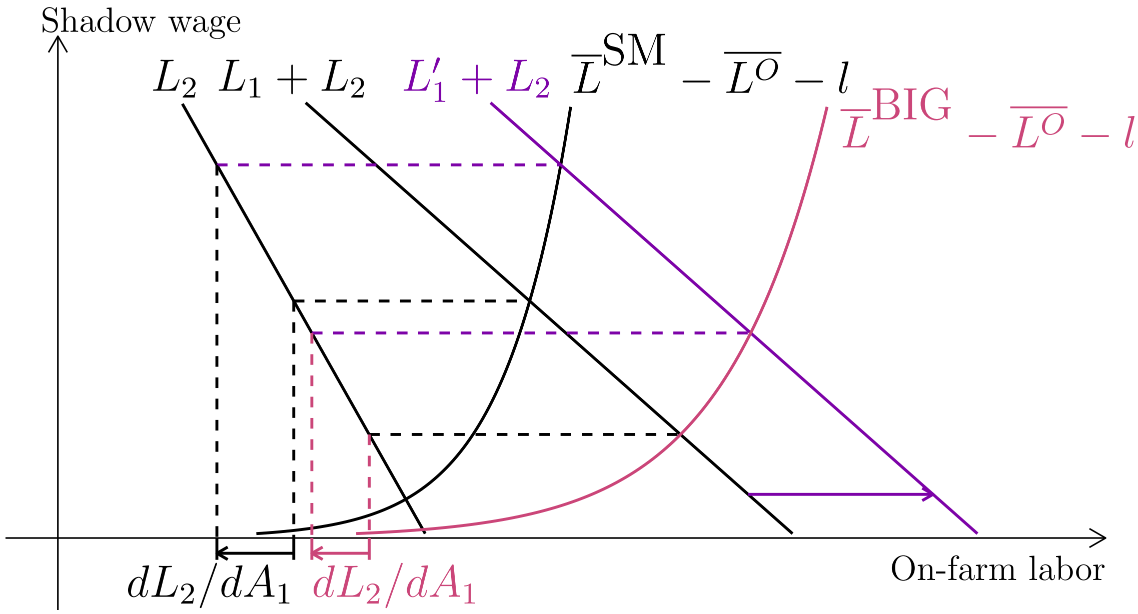


Figure A7: Differential responses to sample plot shock under labor constraints

Notes: Households' labor allocations under a binding off-farm labor constraint are presented in this figure. L_k and l are the household's labor allocation on plot k and choice of leisure, respectively, as a function of the shadow wage, with the argument suppressed. $L_1 + L_2$ is total household on-farm labor demand; if the household's sample plot ($k = 1$) is in the command area ("sample plot shock"), on-farm labor demand shifts out to $L_1' + L_2$. $\overline{L}^{\text{SM}} - \overline{L^O} - l$ is household on-farm labor supply; for large households, on-farm labor supply is shifted out to $\overline{L}^{\text{BIG}} - \overline{L^O} - l$. The shadow wage is determined by the intersection of on-farm labor demand and on-farm labor supply, and labor allocations on the largest other plot are L_2 evaluated at this shadow wage. In this figure, larger households are on a more elastic portion of their on-farm labor supply schedule; as a result, the sample plot shock causes a smaller increase in the shadow wage, and in turn a smaller decrease in labor allocations on the largest other plot (smaller in magnitude dL_2/dA_1).

Appendix C Model

Households have 2 plots, indexed by k: $k = 1$ indicates the sample plot, while $k = 2$ indicates the largest other plot. On each plot k, they have access to a simple production technology $\sigma A_k F_k(M_k, L_k)$ where A_k is plot productivity, M_k is the inputs applied to plot k and L_k is the household labor applied to plot k. The common price and production shock σ is a random variable such that $\sigma \sim \Psi(\sigma), \mathbf{E}[\sigma] = 1$. While this specification assumes a single production function on each plot, we interpret $F_k(M_k, L_k)$ as the envelope of production functions from cultivating different fractions of bananas and horticulture on the dry season; thus we will think of cultivating bananas as optimizing at a low input intensity. Utilizing subscripts to indicate partial derivatives and subsuming arguments we assume $F_{kM} > 0, F_{kL} > 0, F_{kML} > 0, F_{kMM} < 0, F_{kLL} < 0$.

Households have a budget of $\overline{M}$ which, if not utilized for inputs, can be invested in a risk-free asset which appreciates at rate r. In this context, households maximize expected utility over consumption c and leisure l, considering their budget constraint and a labor constraint $\overline{L}$ which is allocated to labor on each plot, leisure, and up to $\overline{L^O}$ units of off-farm labor L^O. Finally, we model irrigation access as an increase in A_1. As we consider the role of each different constraint, we develop the necessary assumptions to produce the results from Section 3: that this increase in A_1 generates an increase in demand for inputs and labor on plot A_1.

Households maximize expected utility

$$\max_{M_1, M_2, L_1, L_2, l, L^O} E[u(c, l)]$$

subject to the constraints enumerated above

$$\begin{aligned} \sigma A_1 F_1(M_1, L_1) + \sigma A_2 F_2(M_2, L_2) + wL^O + r(\overline{M} - M_1 - M_2) &= c \\ M_1 + M_2 &\leq \overline{M} \\ L_1 + L_2 + l + L^O &= \overline{L} \\ L^O &\leq \overline{L^O} \end{aligned}$$

After substituting in the constraints which bind with equality, we derive the following first order conditions

$$(M_k) \quad \left(1 + \tfrac{\text{cov}(\sigma, u_c)}{\mathbf{E}[u_c]}\right) A_k F_{kM} = (1 + \lambda_M) r \tag{A1}$$

$$(L_k) \quad \left(1 + \tfrac{\text{cov}(\sigma, u_c)}{\mathbf{E}[u_c]}\right) A_k F_{kL} = (1 - \lambda_L) w \tag{A2}$$

$$(\ell) \quad \tfrac{\mathbf{E}[u_\ell]}{\mathbf{E}[u_c]} = (1 - \lambda_L) w \tag{A3}$$

Intuitively, the first order conditions for inputs and labor include three parts. First, each contains the marginal product of the factor, $A_k F_{kM}$ and $A_k F_{kL}$ respectively, on the left hand side, and the market price of the factor, r and w respectively, on the right hand side. The second piece, $1 + \frac{\text{cov}(\sigma, u_c)}{\mathbf{E}[u_c]}$, is the ratio of the marginal utility from agricultural production to the marginal utility from certain consumption. This ratio scales down the marginal product of the factor. It is less than 1 because agricultural production is uncertain, and higher in periods in which marginal utility is lower, so $\text{cov}(\sigma, u_c) < 0$. With perfect insurance, $\text{cov}(\sigma, u_c) = 0$, and this term disappears. Without it, however, farmers will underinvest in both inputs and labor relative to the perfect insurance optimum.[A1] Third, there are the Lagrange multipliers associated with the input constraint λ_M and with the labor constraint λ_L, which scale the associated factor prices up and down, respectively.

Proof of Proposition 1 When no constraints bind, the first order conditions simplify to

$$\begin{array}{rll} (M_k) & A_k F_{kM} & = r \\ (L_k) & A_k F_{kL} & = w \\ (\ell) & \frac{u_\ell}{u_c} & = w \end{array}$$

Note that the first order conditions for M_2 and L_2 are functions only of (M_2, L_2), and exogenous (A_2, r, w). Therefore, $\frac{dM_2}{dA_1} = \frac{dL_2}{dA_1} = 0$.

Proof of Proposition 3 **Insurance market failure.** Consider the case when insurance markets fail. To abstract fully from labor supply, we temporarily remove leisure from the model. To further simplify, we drop other inputs from the production function; when the production function is homogeneous in labor and other inputs, this is without loss of generality. Households solve

$$\max_{L_1, L_2} \mathbf{E}[u(c)]$$
$$\sigma(A_1 F_1(L_1) + A_2 F_2(L_2)) - w(L_1 + L_2) + w\overline{L} + r\overline{M} = c$$

[A1] This result does not generically hold in models of agricultural households, as when consumption is separately modeled, households that are net buyers of an agricultural good may overinvest in inputs and labor relative to the perfect insurance optimum (Barrett, 1996). This is unlikely to be first order in our context, as we sampled cultivators and our results are driven by production of commercial crops.

To simplify the analysis, this can be rewritten as the two step optimization problem

$$\begin{aligned}\max_L \mathbf{E}[u(c)]\\ \sigma G(L;A_1) - wL + w\overline{L} + r\overline{M} = c\\ G(L;a) \equiv \max_{L_2} aF_1(L-L_2) + A_2F_2(L_2)\end{aligned}$$

Next, let $\gamma(g,c) = \frac{\mathbf{E}[u_c(\sigma g+c)]}{\mathbf{E}[\sigma u_c(\sigma g+c)]}$; $\gamma \geq 1$ is the ratio of the marginal utility from consumption to the marginal utility from agricultural production. As above, to represent derivatives of G and γ we use subscripts to indicate partial derivatives and subsume arguments. This yields the first order condition

$$(L) \quad G_L - \gamma(G(L;A_1), w(\overline{L}-L) + r\overline{M})w \quad = 0$$

The central intuition for this case can be captured from just the first order condition: $\overline{L}$ and $\overline{M}$ enter symmetrically into the model, so larger households should respond similarly to richer households. If absolute risk aversion decreases sufficiently quickly (e.g., with CRRA preferences), then for sufficiently high levels of consumption $\mathbf{E}[\sigma u_c] = \mathbf{E}[\sigma]\mathbf{E}[u_c] = \mathbf{E}[u_c] \Rightarrow \gamma = 1$. Therefore, sufficiently wealthy or sufficiently large households should not respond to the sample plot shock. Below, we will maintain the assumption that preferences exhibit decreasing absolute risk aversion, and that $\lim_{c\to\infty}\gamma(g,c) = 1$.

Let FOC_L be the left hand side of the first order condition for the utility maximization problem. Then, an application of the implicit function theorem yields $\frac{dL}{dA_1} = -\frac{d\text{FOC}_L/dA_1}{d\text{FOC}_L/dL}$. Evaluating these derivatives yields

$$\begin{aligned}\frac{d\text{FOC}_L}{dL} &= G_{LL} + \gamma_c w^2 - \gamma_g G_L w\\ \frac{d\text{FOC}_L}{dA_1} &= G_{La} - \gamma_G G_a w\\ \frac{dL}{dA_1} &= -\frac{G_{La} - \gamma_g G_a w}{G_{LL} + \gamma_c w^2 - \gamma_g G_L w}\end{aligned}$$

Next, we use the first order condition for constrained production maximization.

Some applications of the envelope theorem yields

$$G_L = A_1 F_{1L}$$
$$G_a = F_1$$

and taking derivatives yields

$$G_{La} = F_{1L}(1 - dL_2/dL)$$
$$G_{LL} = A_1 F_{1LL}(1 - dL_2/dL)$$

Lastly, note that $\frac{dL_2}{dA_1} = \frac{dL_2}{dL}\frac{dL}{dA_1} + \frac{dL_2}{da}\Big|_{a=A_1}$, as the increase in A_1 shifts both arguments to G. Let FOC_{L_2} denote the left hand side of the first order condition for constrained production maximization. Then, applications of the implicit function theorem yield $\frac{dL_2}{dL} = -\frac{d\text{FOC}_{L_2}/dL}{d\text{FOC}_{L_2}/dL_2}$ and $\frac{dL_2}{da} = -\frac{d\text{FOC}_{L_2}/da}{d\text{FOC}_{L_2}/dL_2}$. Calculating the first order condition yields

$$\text{FOC}_{L_2} = -aF_{1L} + A_2 F_{2L}$$

Taking derivatives yields

$$\frac{d\text{FOC}_{L_2}}{da} = -F_{1L}$$
$$\frac{d\text{FOC}_{L_2}}{dL} = -aF_{1LL}$$
$$\frac{d\text{FOC}_{L_2}}{dL_2} = aF_{1LL} + A_2 F_{2LL}$$

and substituting into the expressions we derived above yields

$$\frac{dL_2}{dL} = \frac{aF_{1LL}}{aF_{1LL} + A_2 F_{2LL}}$$
$$\frac{dL_2}{da} = \frac{F_{1L}}{aF_{1LL} + A_2 F_{2LL}}$$

substituting these into our expression for $\frac{dL_2}{dA_1}$, and in turn our expressions for derivatives of G (in the numerator), yields

$$\frac{dL_2}{dA_1} = \frac{-A_1F_{1LL}(G_{La} - \gamma_g G_a w) + F_{1L}(G_{LL} + \gamma_c w^2 - \gamma_g G_L w)}{(A_1F_{1LL} + A_2F_{2LL})(G_{LL} + \gamma_c w^2 - \gamma_g G_L w)}$$
$$= \frac{(F_{1L}w^2)\gamma_c - (F_{1L}^2 - F_{1LL}F_1)A_1 w\gamma_g}{(A_1F_{1LL} + A_2F_{2LL})(G_{LL} + \gamma_c w^2 - \gamma_g G_L w)}$$

To sign this expression, note that the denominator is the product of two second order conditions, for utility maximization and for maximization of production subject to $L_1 = L - L_2$; each of these is negative, so the product is positive. Therefore $\text{sign}(dL_2/dA_1) = \text{sign}((F_{1L}w^2)\gamma_c - (F_{1L}^2 - F_{1LL}F_1)A_1 w\gamma_g)$. Next, note that $F_{1L}w^2 > 0$ and $-(F_{1L} - F_{1LL}F_1)A_1 w < 0$; therefore one sufficient condition for this derivative to be negative is that $\gamma_c < 0$ and $\gamma_g > 0$; in other words, increasing consumption reduces the marginal utility from consumption relative to the marginal utility from agricultural production, and increasing agricultural production increases the marginal utility from consumption relative to the marginal utility from agricultural production. The former generically holds under decreasing absolute risk aversion, while the latter holds under some restrictions; under these restrictions, $\frac{dL_2}{dA_1} < 0$.

For one sufficient restriction, we follow Karlan et al. (2014) and make restrictions on the distribution of σ. We assume that, for some $k > 1$, $\sigma = k$ with probability $\frac{1}{k}$ ("the good state") and $\sigma = 0$ with probability $\frac{k-1}{k}$ ("the bad state"); i.e., there is a crop failure with probability $\frac{k-1}{k}$. Under this assumption, $\gamma = \frac{\mathbf{E}[u_c]}{\mathbf{E}[\sigma u_c]} = \frac{\mathbf{E}[u_c]}{\mathbf{E}[u_c|\sigma=k]}$. Next, define $\overline{R} = -\frac{\mathbf{E}[u_c \frac{u_{cc}}{u_c}]}{\mathbf{E}[u_c]}$ to be the household's (weighted) average risk aversion, and $R_k = -\mathbf{E}[\frac{u_{cc}}{u_c}|\sigma = k]$ to be the household's risk aversion in the good state. Note that by decreasing absolute risk aversion, $R_k < \overline{R}$. Taking derivates of γ and substituting

$$\gamma_c = \frac{\mathbf{E}[u_{cc}]}{\mathbf{E}[\sigma u_c]} - \frac{\mathbf{E}[\sigma u_{cc}]\mathbf{E}[u_c]}{\mathbf{E}[\sigma u_c]^2} = \gamma(R_k - \overline{R}) < 0$$
$$\gamma_g = \frac{\mathbf{E}[\sigma u_{cc}]}{\mathbf{E}[\sigma u_c]} - \frac{\mathbf{E}[\sigma^2 u_{cc}]\mathbf{E}[u_c]}{\mathbf{E}[\sigma u_c]^2} = (k-1)\frac{\mathbf{E}[u_c|\sigma = 0]}{\mathbf{E}[u_c|\sigma = k]}R_k = (k\gamma - 1)R_k > 0$$

Finally, consider the limit as household wealth increases, and assume that agricultural production will not grow infinitely with household wealth; this holds when the marginal product of labor on each plot falls sufficiently quickly and is true of typical decreasing returns to scale production functions. Then, $\lim_{\overline{M}\to\infty} \gamma = 1$ and $\lim_{\overline{M}\to\infty} \gamma_c = \lim_{\overline{M}\to\infty} \gamma_g = 0$, and therefore $\lim_{\overline{M}\to\infty} \frac{dL_2}{dA_1} = 0$. We therefore expect that, heuristically on average, $\frac{d^2 L_2}{dA_1 d\overline{M}} > 0$, as $\frac{dL_2}{dA_1} < 0$ and $\frac{dL_2}{dA_1}$ approaches 0 for large $\overline{M}$. As $\overline{L}$ and $\overline{M}$ enter symmetrically, the same results hold for $\overline{L}$.

Input constraint. When only the input constraint binds, the first order conditions simplify to

$$\begin{aligned} (M_k) \quad & A_k F_{kM} && = (1+\lambda_M) r \\ (L_k) \quad & A_k F_{kL} && = w \\ (\ell) \quad & \tfrac{\mathbf{E}[u_\ell]}{\mathbf{E}[u_c]} && = w \end{aligned}$$

Note that the choice of leisure does not enter into the first order conditions for M_k or L_k. Substituting $M_2 = \overline{M} - M_1$ yields the following system of equations

$$\begin{aligned} A_1 F_{1M}(M_1, L_1) - (1+\lambda_M) r &= 0 \\ A_1 F_{1L}(M_1, L_1) - w &= 0 \\ A_2 F_{2M}(\overline{M} - M_1, L_2) - (1+\lambda_M) r &= 0 \\ A_2 F_{2L}(\overline{M} - M_1, L_2) - w &= 0 \end{aligned}$$

Stack the left hand sides into the vector FOC_M.

Define the Jacobian $J_M \equiv D_{(M_1, L_1, \lambda_M, L_2)} \mathrm{FOC}_M$. Applying the implicit function theorem yields $D_{(A_1)}(M_1, L_1, \lambda_M, L_2)' = -J_M^{-1} D_{(A_1)} \mathrm{FOC}_M$. Taking derivatives and matrix algebra yield

$$\begin{aligned} J_M &= \begin{pmatrix} A_1 F_{1MM} & A_1 F_{1ML} & -r & 0 \\ A_1 F_{1ML} & A_1 F_{1LL} & 0 & 0 \\ -A_2 F_{2MM} & 0 & -r & A_2 F_{2ML} \\ -A_2 F_{2ML} & 0 & 0 & A_2 F_{2LL} \end{pmatrix} \\ D_{(A_1)} \mathrm{FOC}_M &= (F_{1M}, F_{1L}, 0, 0)' \\ \frac{dM_2}{dA_1} &= k_M A_2 F_{2LL} A_1 (F_{1L} F_{1ML} - F_{1M} F_{1LL}) \\ \frac{dL_2}{dA_1} &= -k_M A_2 F_{2ML} A_1 (F_{1L} F_{1ML} - F_{1M} F_{1LL}) \end{aligned}$$

where k_M is positive.[A2] As $F_{2LL} < 0$, $\text{sign}\left(\frac{dM_2}{dA_1}\right) = -\text{sign}\left(F_{1L}F_{1ML} - F_{1M}F_{1LL}\right)$. This is negative whenever productivity growth on plot 1 would cause optimal input allocations, holding fixed the shadow price of inputs, to increase on plot 1 – this is mechanical, as total input allocations are constrained at $\overline{M}$. Similarly, $\text{sign}\left(\frac{dL_2}{dA_1}\right) = \text{sign}(F_{2LM})\text{sign}\left(\frac{dM_2}{dA_1}\right)$. The labor response and input response on the second plot have the same sign whenever labor and inputs are complements on the second plot.

Proof of Proposition 4 When only the labor constraint binds, the first order conditions simplify to

$$\begin{aligned}
(M_k) \quad & A_k F_{kM} && = r \\
(L_k) \quad & A_k F_{kL} && = (1-\lambda_L)w \\
(\ell) \quad & \tfrac{u_\ell}{u_c} && = (1-\lambda_L)w
\end{aligned}$$

Substituting $\ell = \overline{L} - L^O - L_1 - L_2$ and $L^O = \overline{L^O}$, and some rearranging and substitutions yield

$$\begin{aligned}
A_1F_{1M}(M_1, L_1) - r = 0 \\
A_1F_{1L}(M_1, L_1) - (1+\lambda_L)w = 0 \\
A_2F_{2M}(M_2, L_2) - r = 0 \\
A_2F_{2L}(M_2, L_2) - (1+\lambda_L)w = 0 \\
u_\ell\left(\sum_{k\in\{1,2\}} A_kF_k(M_k, L_k) + r(\overline{M} - M_1 - M_2) + w\overline{L^O}, \overline{L} - \overline{L^O} - L_1 - L_2\right) - \\
(1+\lambda_L)wu_c\left(\sum_{k\in\{1,2\}} A_kF_k(M_k, L_k) + r(\overline{M} - M_1 - M_2) + w\overline{L^O}, \overline{L} - \overline{L^O} - L_1 - L_2\right) = 0
\end{aligned}$$

Stack the left hand sides into the vector FOC_L.

Additionally, it will be convenient to define the following derivatives of on-farm

[A2] $k_M = -\frac{1}{(A_1F_{1LL})A_2^2(F_{2MM}F_{2LL}-F_{2ML}^2)+(A_2F_{2LL})A_1^2(F_{1MM}F_{1LL}-F_{1ML}^2)}$. We make standard assumptions required for unconstrained optimization; second order conditions for unconstrained optimization imply k_M is positive.

labor demand on plot k, LD_k, with respect to the shadow wage w^* and productivity A_k, on-farm input demand on plot k, MD_k, with respect to productivity A_k, and on-farm labor supply, LS, with respect to the shadow wage w^* and consumption (through shifts to wealth) c. Let

$$\begin{aligned}
\text{LD}_{kw^*} &= \frac{A_k F_{kMM}}{A_k^2(F_{kMM}F_{kLL} - F_{kML}^2)} \\
\text{LD}_{kA_k} &= \frac{A_k F_{kM} F_{kML} - A_k F_{kL} F_{kMM}}{A_k^2(F_{kMM}F_{kLL} - F_{kML}^2)} \\
\text{MD}_{kA_k} &= \frac{A_k F_{kL} F_{kML} - A_k F_{kM} F_{kLL}}{A_k^2(F_{kMM}F_{kLL} - F_{kML}^2)} \\
\text{LS}_{w^*} &= -\frac{u_c}{u_{\ell\ell} - (1+\lambda_L) w u_{c\ell}} \\
\text{LS}_c &= -\frac{u_{c\ell} - (1+\lambda_L) w u_{cc}}{u_{\ell\ell} - (1+\lambda_L) w u_{c\ell}}
\end{aligned}$$

We make standard assumptions required for unconstrained optimization; these imply LD_{kw^*} is negative (labor demand decreasing in shadow wage), and LS_{w^*} is positive (labor supply increasing in shadow wage). We further assume LD_{kA_k} and MD_{kA_k} are positive (labor demand and input demand are increasing in productivity); an additional sufficient assumption for this is that F is homogeneous. We further assume LS_c is negative (labor supply is decreasing in wealth); an additional sufficient assumption for this is that u is additively separable in c and ℓ.

Next, define the Jacobian $J_L \equiv D_{(M_1,L_1,M_2,L_2,\lambda_L)}\text{FOC}_L$. Taking derivatives and matrix algebra yield

$$J_L = \begin{pmatrix} A_1F_{1MM} & A_1F_{1ML} & 0 & 0 & 0 \\ A_1F_{1ML} & A_1F_{1LL} & 0 & 0 & -w \\ 0 & 0 & A_2F_{2MM} & A_2F_{2ML} & 0 \\ 0 & 0 & A_2F_{2ML} & A_2F_{2LL} & -w \\ \frac{d\text{FOC}_{L,\ell}}{dM_1} & \frac{d\text{FOC}_{L,\ell}}{dL_1} & \frac{d\text{FOC}_{L,\ell}}{dM_2} & \frac{d\text{FOC}_{L,\ell}}{dL_2} & -wu_c \end{pmatrix}$$

$$\frac{d\text{FOC}_{L,\ell}}{dM_1} = A_1F_{1M}(u_{c\ell} - (1+\lambda_L)wu_{cc})$$

$$\frac{d\text{FOC}_{L,\ell}}{dL_1} = A_1F_{1L}(u_{c\ell} - (1+\lambda_L)wu_{cc}) - (u_{\ell\ell} - (1+\lambda_L)wu_{c\ell})$$

$$\frac{d\text{FOC}_{L,\ell}}{dM_2} = A_2F_{2M}(u_{c\ell} - (1+\lambda_L)wu_{cc})$$

$$\frac{d\text{FOC}_{L,\ell}}{dL_2} = A_2F_{2L}(u_{c\ell} - (1+\lambda_L)wu_{cc}) - (u_{\ell\ell} - (1+\lambda_L)wu_{c\ell})$$

Applying the implicit function theorem yields $D_{(A_1)}(M_1, L_1, M_2, L_2, \lambda_L)' = -J_L^{-1}D_{(A_1)}\text{FOC}_L$. Some further algebra and taking derivatives, and substitution, yield

$$D_{(A_1)}\text{FOC}_L = (F_{1M}, F_{1L}, 0, 0, (u_{c\ell} - (1+\lambda_L)wu_{cc})F_1)'$$

$$\frac{dL_2}{dA_1} = \text{LD}_{2w^*}\frac{\text{LD}_{1A_1} - \text{LS}_c(F_{1M}\text{MD}_{1A_1} + F_{1L}\text{LD}_{1A_1} + F_1)}{\text{LS}_{w^*} - (\text{LD}_{1w^*} + \text{LD}_{2w^*}) - \text{LS}_c(\text{LD}_{1A_1} + \text{LD}_{2A_2})}$$

$$\frac{dL_2}{d\bar{L}} = \text{LD}_{2w^*}\frac{1}{\text{LS}_{w^*} - (\text{LD}_{1w^*} + \text{LD}_{2w^*}) - \text{LS}_c(\text{LD}_{1A_1} + \text{LD}_{2A_2})}$$

$$\frac{dL_2}{d\bar{M}} = \text{LD}_{2w^*}\frac{r\text{LS}_c}{\text{LS}_{w^*} - (\text{LD}_{1w^*} + \text{LD}_{2w^*}) - \text{LS}_c(\text{LD}_{1A_1} + \text{LD}_{2A_2})}$$

$\frac{dL_2}{dA_1} < 0$; for interpretation, note that this expression is the derivative of labor demand on plot 2 with respect to the shadow wage, times the effect of the shock to A_1 on the shadow wage. The numerator of the latter is the effect the shock on negative residual labor supply through direct effects (LD_{1A_1}) and wealth effects, including through adjustments of labor and inputs ($-\text{LS}_c(F_{1M}\text{MD}_{1A_1} + F_{1L}\text{LD}_{1A_1} + F_1)$). The denominator of the latter is the derivative of residual labor supply with respect to the shadow wage, adjusted for wealth effects ($\text{LS}_{w^*} - (\text{LD}_{1w^*} + \text{LD}_{2w^*}) - \text{LS}_c(\text{LD}_{1A_1} + \text{LD}_{2A_2})$).

The signs of $\frac{d^2L_2}{d\bar{L}dA_1}$ and $\frac{d^2L_2}{d\bar{M}dA_1}$ are ambiguous. However, unlike the cases of input market failures or insurance market failures, here these second derivatives may have opposite signs. To see one example of this, consider a case where on-farm labor and input demands are approximately linear in the shadow wage and productivity, and on-farm labor supply is approximately linear in consumption, but exhibits meaningful curvature with respect to the shadow wage. In this case, $\text{sign}(\frac{d^2L_2}{d\bar{L}dA_1}) = \text{sign}\left(\frac{d}{d\bar{L}}\text{LS}_{w^*}\right)$ and $\text{sign}(\frac{d^2L_2}{d\bar{L}dA_1}) = \text{sign}\left(\frac{d}{d\bar{M}}\text{LS}_{w^*}\right)$. To focus on one case, larger households are less responsive to the A_1 shock ($\frac{d^2L_2}{d\bar{L}dA_1} > 0$) if and only if they are on a more elastic portion of their labor supply curve ($\frac{d}{d\bar{L}}\text{LS}_{w^*} > 0$). That larger households, with more labor available for agriculture, or poorer households, who likely have fewer productive opportunities outside agriculture, would be on a more elastic portion of their labor supply curve is consistent with proposed models of household labor supply dating back to Lewis (1954). This motivates the prediction we focus on: that larger households should be less responsive to the A_1 shock, and richer households should be more responsive to the A_1 shock.

We present this case graphically in Appendix Figure A7.